Time Wavenumber Formulation for Propagation of Acoustic Pressure Fields

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Abstract

The wavenumber method combined with the FFT algorithm is established as a more efficient tool than the Rayleigh integral method for evaluating harmonic acoustic fields from planar radiators whose sound emission is assumed to be stationary. However, this method is not adapted when analysing non-harmonic acoustic fields. Therefore this work deals with the calculation of non-stationary acoustic fields using a time domain impulse response formulation of the problem in the wavenumber domain. Some explanations to reach the formulation are given in the paper. Numerical simulations are also reported to validate the formulation. These simulations concern sources composed of monopoles whose sound emission fluctuates in time.

1 Introduction

The association of the wavenumber method with the FFT algorithm is regarded as a more efficient tool than the Rayleigh integral method for determining harmonic acoustic fields from planar radiators [1]. Nevertheless the limitation of this method is that it does not match to the determination of acoustic fields whose sound emission is non-stationary and not limited in time. Therefore the purpose of this work is to provide a method of reconstruction of non-harmonic acoustic fields using a time domain impulse response formulation in the wavenumber domain. Explanations about the theory to reach the impulse response are presented in the next section.

2 Theory

The sources are located in the region $z < z_A$ (see the geometry of interest in Figure 1). From the formulation in the wavenumber domain, the time-dependant wavenumber spectrum $P(K_x, K_y, z_R, t)$ in a forward plane $z = z_R$ (or propagation plane) can be calculated by convolving each component of the time-dependant wavenumber spectrum $P(K_x, K_y, z_A, t)$ in a measurement plane $z = z_A$ ($z_A < z_R$) with a time domain impulse response h :

$$P(K_x, K_y, z_R, t) = P(K_x, K_y, z_A, t) * h(K_x, K_y, z_R - z_A, t) \tag{1}$$

To obtain this result the acoustic propagation in the wavenumber domain described by the temporal wave equation in plane geometry is considered [2] :

$$\Delta p(x, y, z, t) - \frac{1}{c^2}\frac{\partial^2 p(x, y, z, t)}{\partial t^2} = 0 \tag{2}$$

A two-dimensional Fourier transform is applied to this equation that leads to a differential equation written in a Laplace formalism. The resolution of this equation gives the expression of the impulse response h where

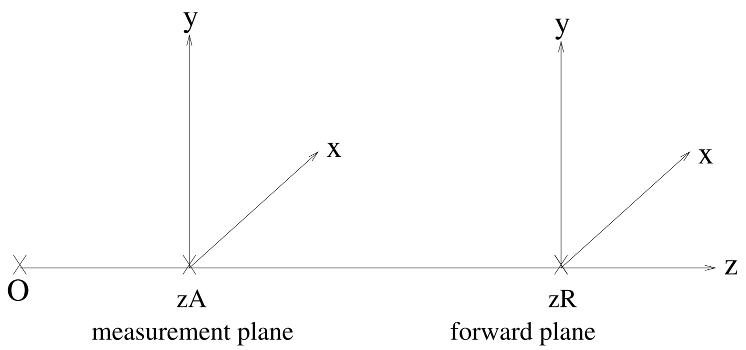

Figure 1: The geometry of interest.

J_1 is the first order Bessel function and $dz = z_R - z_A$ the distance between the measurement plane $z = z_A$ and the propagation plane $z = z_R$ [3] :

$$h(K_x, K_y, dz, t) = \delta\left(t - \frac{dz}{c}\right) - dz\sqrt{K_x^2 + K_y^2}\frac{J_1\left(c\sqrt{K_x^2 + K_y^2}\sqrt{t^2 - \frac{dz^2}{c^2}}\right)}{\sqrt{t^2 - \frac{dz^2}{c^2}}}\Gamma\left(t - \frac{dz}{c}\right) \quad (3)$$

$\delta(t)$ is the dirac function and Γ is the Heaviside function defined by :

$$\Gamma(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } t \geq 0 \end{cases}$$

In (1), by replacing h with its expression from (3), the time-dependant wavenumber spectrum in the forward plane $P(K_x, K_y, z_R, t)$ is calculated :

$$\begin{aligned} P(K_x, K_y, z_R, t) &= P(K_x, K_y, z_A, t - \frac{dz}{c}) \\ &-P(K_x, K_y, z_A, t) * \left[dz\sqrt{K_x^2 + K_y^2}\frac{J_1\left(c\sqrt{K_x^2+K_y^2}\sqrt{t^2-\frac{dz^2}{c^2}}\right)}{\sqrt{t^2-\frac{dz^2}{c^2}}}\Gamma\left(t - \frac{dz}{c}\right)\right] \end{aligned} \quad (4)$$

Then the two-dimensional inverse Fourier transform in space domain provides the instantaneous spatial pressure in the forward plane $p(x, y, z_R, t)$.

3 Time domain results

3.1 Numerical setup

First a source composed of two monopoles at the positions A_1 $(0.2, 0.8, 0)$ and A_2 $(0.6, 0.4, 0)$ is implemented considering the geometry in Figure 1. Each monopole radiates a signal whose expression is :

$$s(t) = sin(2\pi ft)e^{-\lambda t}$$

The signals are attenuated by a decreasing exponential function to be sure of the non-stationary emission of the source. The modulation frequency of the two signals is equal to 600 Hz and the parameter λ is equal to 300 for the signal radiated by the monopole located on A_1 and equal to 200 for the one radiated by the monopole on A_2. The simulation of the acquisition of the pressure field is done by a microphones array located in the measurement plane $z = z_A$ with $z_A = 0.05$ m. The resulting pressure field is then forward projected to the plane $z = z_R$ with $z_R = 0.1$ m. A reference pressure field directly simulated in the plane $z_R = 0.1$ is also calculated to make a comparison. The simulation involves input pressure signals sampled at a sampling frequency f_e.

3.2 Enlargement of the array and oversampling

To improve results, the enlargement of the array and the oversampling are tested to avoid errors associated with the transition of the theory to the numerical approach.

- **Enlargement of the array**
  In theory the measurement plane is assumed to be infinite in extent. In practice, the data recorded from the measurement plane can only be sampled on a surface of finite extent. Therefore to be closer to the theory and to avoid processing errors resulting from the finite size of the array, the measurement plane must be sufficiently larger than the source.

- **Oversampling**
  A second modification is done to improve results. It consists in increasing the sampling frequency f_e. Indeed, the input signals were oversampled to avoid aliasing effects because the impulse response h is not band-limited in the frequency domain.

The effects of the oversampling and of the enlargement of the array can be observed in Figures 2 and 3 where time pressure signals are plotted. These signals are calculated by applying a two-dimensional inverse Fourier transform to equation (4). The time pressure signal $p(0.6, 0.4, z_R, t)$ from location $(0.6, 0.4)$ in the plane $z = z_R$ (in front of the monopole A_2) is plotted in Figures 2 and 3. Two indicators T_1 and T_2, calculated in relation to the reference are given in the caption of the figures. T_1 provides information on the phase and the form of the signals and T_2 on the amplitude of the signals :

$$T_1 = \frac{< s_r(t) s_f(t) >}{\sqrt{< s_f^2(t) >< s_r^2(t) >}}$$

$$T_2 = \sqrt{\frac{< s_f^2(t) >}{< s_r^2(t) >}}$$

$s_r(t)$ is the reference signal and $s_f(t)$ is the forward projected signal ($<>$ is the mean).

As it can be observed in Figure 2, the oversampling seems to be profitable to the forward projecting method as well on the phase than on the amplitude of the forward projected signal because T_1 and T_2 are better with $f_e = 40000$ Hz than with $f_e = 10000$ Hz. In the same way, the results are improved with the enlargement of the array (see Figure 3). Nevertheless, the oversampling seems to be more influent on the phase and the form of the signals than the enlargement of the array (see the parameter T_1).

3.3 Filtering of the impulse response

The filtering of the impulse response is performed to avoid errors due to the not limited frequency band of its Fourier transform. The principle of the filtering is the following : first the input signals are sampled at a sampling frequency $f_e = 10000$ Hz. The impulse response is sampled at a sampling frequency $f_e^{'} = 40000$ Hz and filtered using a two order low-pass butterworth filter. Then the filtered impulse response is undersampled to $f_e = 10000$ Hz to be convolved with the input signals. The best result is obtained for a butterworth filter with a cutoff frequency $f_c = 3400$ Hz. This result can be observed in Figure 4 where three signals are represented : the reference signal, the forward projected signal after filtering the impulse response and the forward projected signal after directly sampling the impulse response at $f_e = 10000$ Hz. Filtering the impulse response improves results. It is shown in Figure 4 by comparing the signal obtained without filtering the impulse response to the signal obtained after filtering the impulse response.

The filtering of the impulse response is tested on an other acoustic source. It is composed of one monopole centered in relation to the array. The monopole radiates a Morlet wavelet signal i.e. a signal written as

$s(t) = cos(2\pi ft)e^{-\lambda t^2/2}$. The result is plotted in Figure 5. The cutoff frequency of the two order low-pass butterworth filter is $f_c = 6000$ Hz. This time, filtering the impulse response is beneficial to the amplitudes (by looking at the indicator T_2) but leads to a phasing between the reference signal and the forward projected signal.

4 Conclusion

The method of the forward propagation of pressure signals from a non-stationary source was presented. Some processings were performed to improve the results of this propagation : the oversampling, the enlargement of the acquisition array and the filtering of the impulse response used in the method of interest. These processings were done to avoid numerical errors as the aliasing distorsion.

A direct prolongation of this work could be the synthesis of a numerical filter from the impulse response h. In the future, the continuation of the work would be the development of a method based on the inverse of the formulation mentioned above i.e. the backward propagation of time evolving acoustic pressure fields.

References

[1] E.G. WILLIAMS and J.D. MAYNARD. Numerical evaluation of the Rayleigh integral for planar radiators using the FFT. J. Acoust. Soc. Am. **72**(6) (1982), 2020-2030

[2] E.G. WILLIAMS. Fourier Acoustics, Academic Press, 1999

[3] M. FORBES, S. LETCHER and P. STEPANISHEN. A wave vector, time-domain method of forward projecting time-dependent pressure fields. J. Acoust. Soc. Am. **90**(5) (1991), 2782-2793

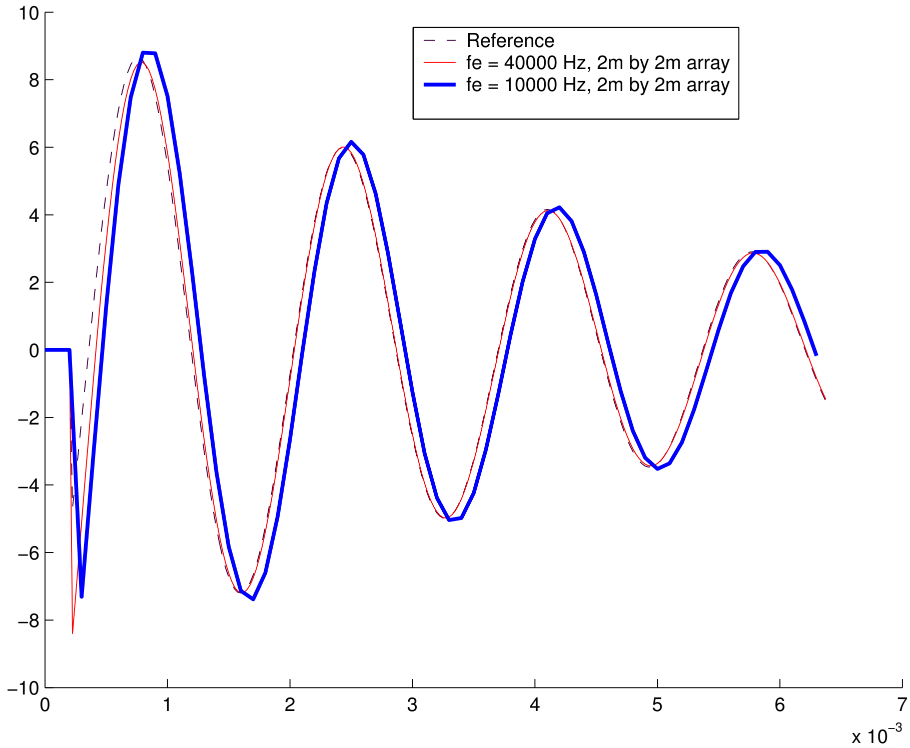

Figure 2: Effects of the oversampling on the time dependant pressure at position (0.6,0.4,0.1). The size of the array is 2 m by 2 m. At $f_e = 10000$ Hz, $T_1 = 0.9333$ and $T_2 = 1.0517$. At $f_e = 40000$ Hz, $T_1 = 0.9894$ and $T_2 = 0.9943$.

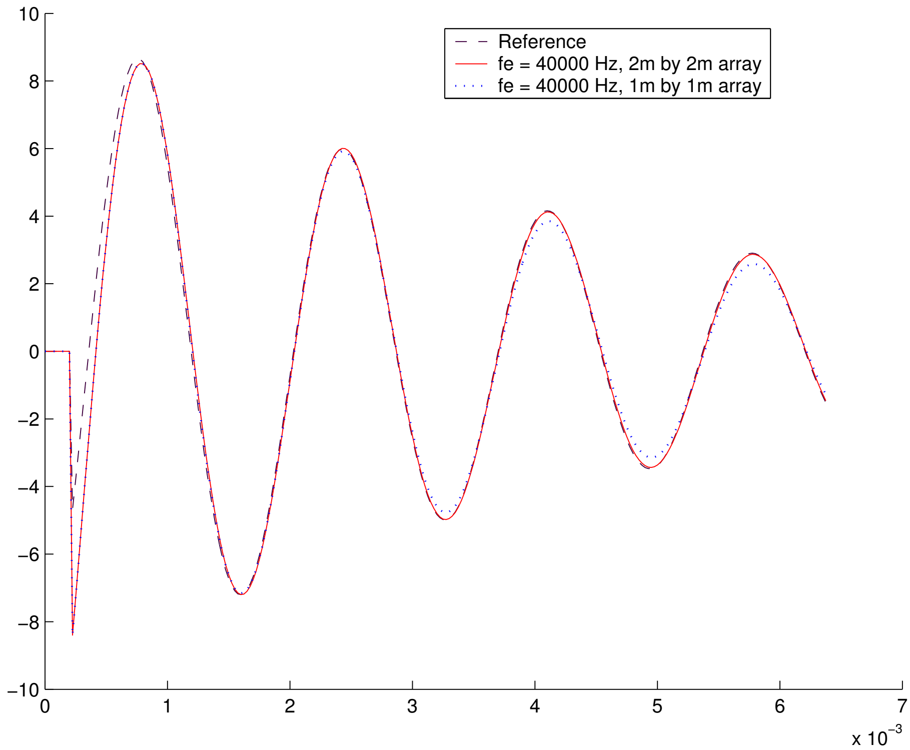

Figure 3: Effects of the decreasing of the array on the time dependant pressure at the position (0.6,0.4,0.1). The sampling frequency f_e is equal to 40000 Hz. For a 1 m by 1 m array, $T_1 = 0.9884$ and $T_2 = 0.9744$. For a 2 m by 2 m array, $T_1 = 0.9894$ and $T_2 = 0.9943$

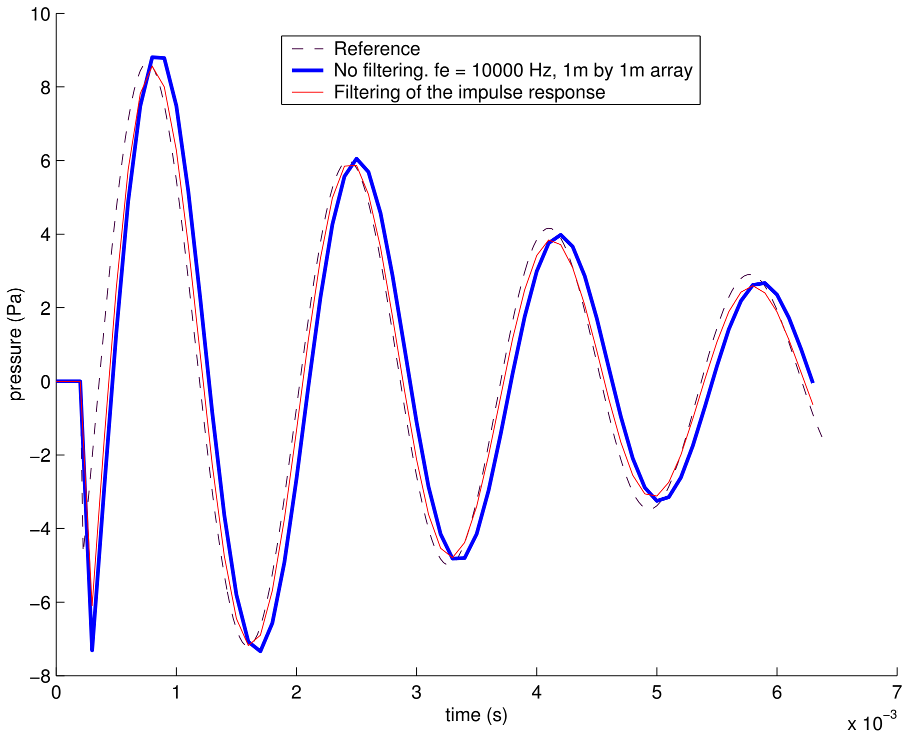

Figure 4: Effects of the filtering of the impulse response on the time dependant pressure at position (0.6,0.4,0.1). The size of the array is 1 m by 1 m and the sampling frequency f_e is equal to 10000 Hz. For the signal obtained without filtering the impulse response, $T_1 = 0.9293$ and $T_2 = 1.0323$ and for the one obtained after filtering the impulse response, $T_1 = 0.9780$ and $T_2 = 0.9885$.

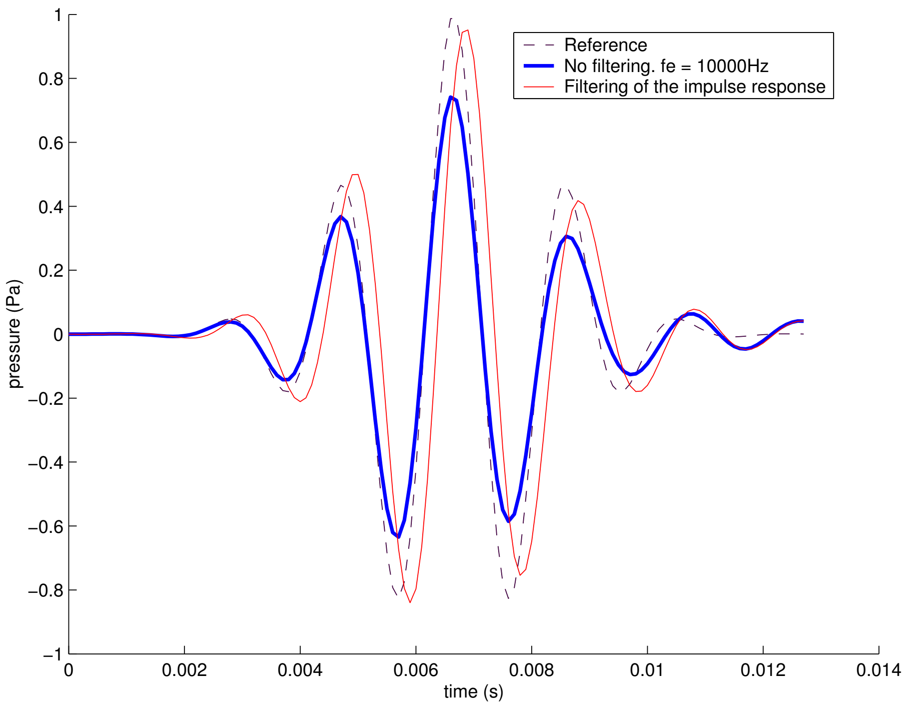

Figure 5: Time dependant pressure in front of a monopole which radiates a Morlet wavelet signal. The size of the array is 1 m by 1 m and the sampling frequency f_e is equal to 10000 Hz. For the signal obtained without filtering of the impulse response, $T_1 = 0.9940$ and $T_2 = 0.79$ and for the one obtained after filtering of the impulse response, $T_1 = 0.7656$ and $T_2 = 0.9745$.