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Time Varying Forward Projection Using Wavenumber Formulation

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Abstract [406] The association of the wavenumber method with the FFT algorithm is regarded as a more efficient tool than the Rayleigh integral method for determining harmonic acoustic fields from planar radiators. Nevertheless the limitation of this method is that it does not match to the determination of acoustic fields whose sound emission is non-stationary and not limited in time. Therefore the purpose of this work is to provide a method of reconstruction of non-harmonic acoustic fields using in the wavenumber domain a time domain impulse response formulation. Explanations about the theory to reach the impulse response are presented in the paper. A real-time prediction of the acoustic wave propagation in the near-field of vibrating structure or in waveguide for applications in active control is given by the formulation. Numerical simulations are also reported to validate the method. The topic of the simulations is the analysis of non-stationary sound sources.

1 INTRODUCTION

The wavenumber method using the FFT algorithm has proven to be a more efficient tool than the Rayleigh integral method for calculating harmonic acoustic fields from planar radiators [1]. However, this method is not adapted when considering long non-stationary signals. Therefore the aim of this work is to provide a relevant reconstruction of non-stationary acoustic fields in real-time using a time domain impulse response formulation of the problem in the wavenumber domain.

The first section of the paper contains explanations to reach the formulation. The last section deals with numerical results concerning non-stationary sources composed of monopoles whose sound emission fluctuates in time.

2 THEORY

2.1 Principle and geometry of interest

Figure (1) shows the geometry of the problem, the sources are assumed to be in the region $z < z_A$. The principle of the method consists in the fact that the time-dependant wavenumber spectrum $P(K_x, K_y, z_R, t)$ in a forward plane $z = z_R$ can be obtained by the convolution of each

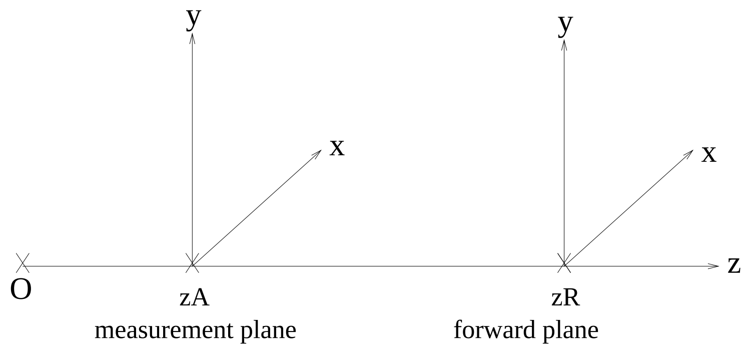

Figure 1: *The geometry of interest.*

component of the time-dependant wavenumber spectrum $P(K_x, K_y, z_A, t)$ in a measurement plane $z = z_A$ ($z_A < z_R$) with a time domain impulse response h :

$$P(K_x, K_y, z_R, t) = P(K_x, K_y, z_A, t) * h(K_x, K_y, z_R - z_A, t) \tag{1}$$

K_x and K_y are the wavenumber coordinates and $*$ denotes the convolution product.

2.2 Calculation of the impulse response

The acoustic propagation described by the wave equation in plane geometry is considered [2] :

$$\Delta p(x, y, z, t) - \frac{1}{c^2}\frac{\partial^2 p(x, y, z, t)}{\partial t^2} = 0 \tag{2}$$

Then a two-dimensional spatial Fourier transform is applied to this equation that yields a differential equation written in a Laplace formalism. The resolution of this equation provides the expression of the impulse response in equation (1) with J_1 the first order Bessel function and $dz = z_R - z_A$ the distance beetween the measurement plane $z = z_A$ and the forward plane $z = z_R$ [3] :

$$h(K_x, K_y, dz, t) = \delta\left(t - \frac{dz}{c}\right) - dz\sqrt{K_x^2 + K_y^2}\frac{J_1\left(c\sqrt{K_x^2 + K_y^2}\sqrt{t^2 - \frac{dz^2}{c^2}}\right)}{\sqrt{t^2 - \frac{dz^2}{c^2}}}\Gamma\left(t - \frac{dz}{c}\right) \tag{3}$$

$\delta(t)$ is the dirac function and Γ is the Heaviside function defined by :

$$\Gamma(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } t \geq 0 \end{cases}$$

The impulse response h can also be written as below with $K_r = \sqrt{K_x^2 + K_y^2}$, $\tau = dz/c$ and $\Omega_r = cK_r$:

$$h(\Omega_r, \tau, t) = \delta\left(t - \tau\right) - \tau\Omega_r^2\frac{J_1\left(\Omega_r\sqrt{t^2 - \tau^2}\right)}{\Omega_r\sqrt{t^2 - \tau^2}}\Gamma\left(t - \tau\right) \tag{4}$$

$\tau = dz/c$ is the delay corresponding to the propagation distance dz and Ω_r is the transition pulsation.

2.3 Calculation of the instantaneous spatial pressure

The time-dependant wavenumber spectrum in the forward plane $P(K_x, K_y, z_R, t)$ is obtained by considering equations (1) and (3) :

$$P(K_x, K_y, z_R, t) = P(K_x, K_y, z_A, t - \frac{dz}{c}) \\ -P(K_x, K_y, z_A, t) * \left[dz\sqrt{K_x^2 + K_y^2} \frac{J_1\left(c\sqrt{K_x^2+K_y^2}\sqrt{t^2-\frac{dz^2}{c^2}}\right)}{\sqrt{t^2-\frac{dz^2}{c^2}}} \Gamma\left(t - \frac{dz}{c}\right) \right] \quad (5)$$

Then the two-dimensional inverse Fourier transform in space domain leads to the instantaneous spatial pressure in the forward plane $p(x, y, z_R, t)$.

3 NUMERICAL RESULTS

3.1 Numerical setup

A source composed of two monopoles at the positions A_1 $(0.75, 0.25, 0)$ and A_2 $(0.75, 0.75, 0)$ was implemented considering the geometry in figure 1. Each monopole radiates a signal whose expression is :

$$s(t) = sin(2\pi f t)e^{-\lambda t}$$

The signals are attenuated by a decreasing exponential function to be sure of the non-stationary condition of the source. The frequency of the two signals is equal to 600 Hz and the parameter λ is equal to 300 for the signal radiated by the monopole located on A_1 and equal to 100 for the one radiated by the monopole on A_2. The simulation of the acquisition of the pressure field is done by a microphone array located in the measurement plane $z = z_A$ with $z_A = 0.05$ m. The resulting pressure field is then forward projected to the plane $z = z_R$ with $z_R = 0.1$ m. A reference spatial pressure field $R(x, y)$ is obtained by a direct acquisition in the plane $z = z_R$ to compare to the forward projected pressure field. The reference spatial pressure field in the plane $z = 0.1$ m at $t = 2.9$ ms is plotted on Figure 2. For each forward projected pressure field $F(x, y)$, two indicators, S_1 and S_2, are given in the title of the figure. S_1 provides information about the shape and S_2 about the amplitude of the forward projected pressure field in relation to the reference pressure field (<> denotes the mean) :

$$S_1 = \frac{< R(x,y)F(x,y) >}{\sqrt{< |F(x,y)|^2 >< |R(x,y)|^2 >}}$$

$$S_2 = \sqrt{\frac{< |F(x,y)|^2 >}{< |R(x,y)|^2 >}}$$

3.2 Improvement of the results

To obtain the forward projected pressure field on Figure 3, the input pressure signals are sampled with a sampling frequency $f_e = 10000$ Hz. The microphone array is composed of 17 by 17 microphones equally spaced by 0.0625 m i.e. a 1 m by 1 m array. The resulting amplitude is abnormally superior to the reference one and there is some geometric distorsion. Moreover, because of the truncation of the measurement plane, some areas with high amplitudes which were not expected appear near to the borderlines of the plane.

ENLARGEMENT OF THE ARRAY To improve results, the enlargement of the array was tested. In theory the measurement plane is assumed to be infinite in extent. In practice, the data recorded on the measurement plane (the hologram) can be only sampled on a surface of finite extent. Therefore to be closer to the theory and to avoid processing errors resulting from the finite size of the hologram, the measurement plane must be sufficiently larger than the source. The forward projected pressure field on Figure 4 results from the acquisition by a 2 m by 2 m array. The amplitude on this figure is closer to the amplitude reference ($S_2 = 1.0453$ vs $S_2 = 1.1659$ for Figure 3) and the shapes are better ($S_1 = 0.9664$ vs $S_1 = 0.9606$ for Figure 3)).

OVERSAMPLING A second modification was used to try to improve results : the increasing of the sampling frequency f_e. Indeed the input signals were oversampled to avoid aliasing effects and because the impulse response h is not band-limited in the frequency domain. The forward projected pressure field after oversampling ($f_e = 40000$ Hz) is plotted on Figure 5. The improvement of the results can be appreciated again. The enlargement of the array is more influent on the amplitudes than the oversampling (see the parameter S_2). On the other hand, the oversampling seems to be more influent on the shape of the radiating fields than the enlargement of the array (see the parameter S_1). Figure 6 shows the forward projected pressure field after the enlargement of the array and the oversampling. The agreement between the results from the direct calculation and those from the forward propagation is good, S_1 and S_2 get better again.

The effect of the oversampling can be observed also on the figures 7 and 8 where time pressure signals are plotted, those signals are calculated by applying a two-dimensional inverse Fourier transform to the equation (5). On Figure 7 the signal $p(0.75, 0.75, 0.1, t)$ is plotted and on figure 8 the signal $p(0.75, 0.5, 0.1, t)$ i.e. the pressure signals respectively at the locations $(0.75, 0.75)$ (in front of the monopole A_2) and $(0.75, 0.5)$ (in the interferences zone between the two monopoles) according to the time in the plane $z = z_R$. Each figure shows a reference signal resulting from the direct acquisition in the plane $z = z_R$, a forward projected signal calculated from an input signal sampled with $f_e = 10000$ Hz and a forward projected signal calculated from an input signal sampled with $f_e = 40000$ Hz. T_1 is calculated in relation to the reference to give information on the phase and the form of the signals and T_2 on the amplitude of the signals with $s_r(t)$ the reference signal and $s_f(t)$ the forward projected signal :

$$T_1 = \frac{< s_r(t) s_f(t) >}{\sqrt{< |s_f(t)|^2 >< |s_r(t)|^2 >}}$$

$$T_2 = \sqrt{\frac{< |s_f(t)|^2 >}{< |s_r(t)|^2 >}}$$

The oversampling seems to be profitable to the forward projecting method as well on the phase than on the amplitude of the forward projected signals because for the two measurements points $(0.75, 0.75)$ and $(0.75, 0.5)$ in the forward plane, T_1 and T_2 are better with $f_e = 40000$ Hz than with $f_e = 10000$ Hz.

4 CONCLUSION

The theory of the forward pojection of time dependant pressure fields was exposed. Numerical results have shown the beneficial effects of the oversampling and of the enlargement of the array on the results. After those two processings, the coherence between the resulting forward projected and the reference pressure fields is fairly good.

In the future, the synthesis of a numerical filter associated with the impulse response h would lead to develop the formulation, it could be tested on practical cases. At last a method resulting from the inverse of the formulation described above, in other words the backward propagation of time varying acoustic fields would be developed.

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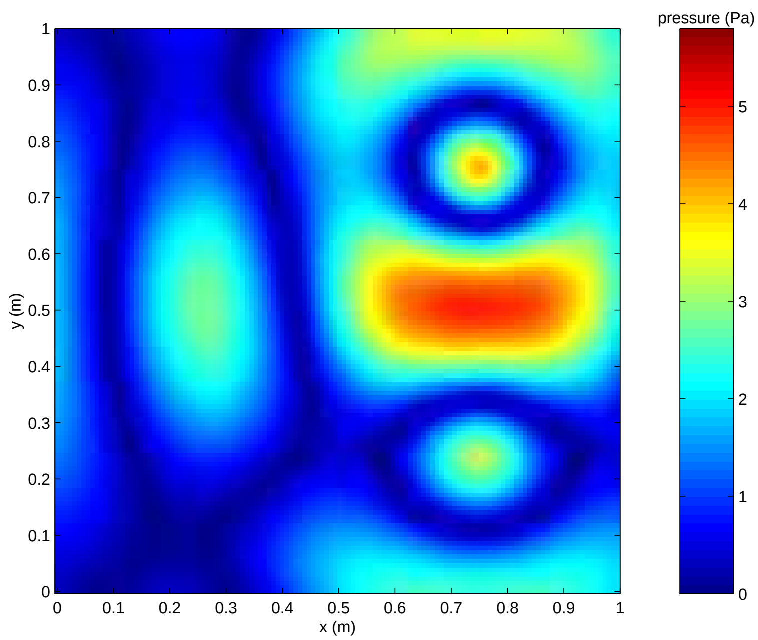

Figure 2: *Reference spatial pressure field (Pa) in the plane* $z = 0.1$ *m at* $t = 2.9$ *ms (modulus).*

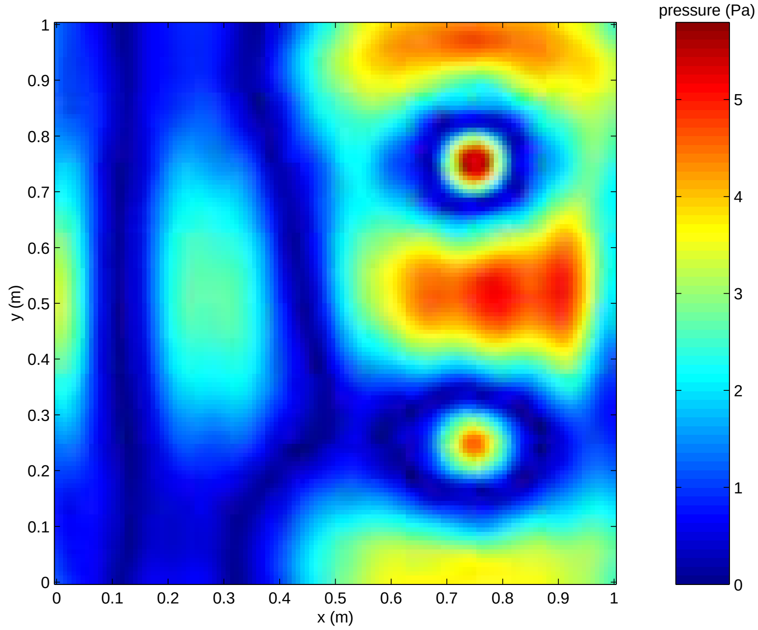

Figure 3: *Forward spatial pressure field (Pa) in the plane* $z = 0.1$ *m at* $t = 2.9$ *ms computed from the spatial pressure field in the plane* $z = 0.05$ *m (modulus).* $S_1 = 0.9606$; $S_2 = 1.1659$.

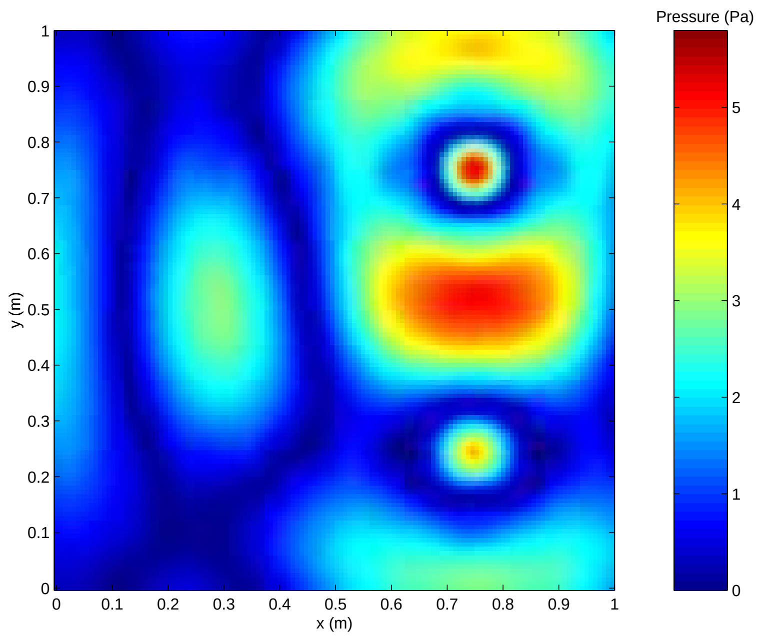

Figure 4: *Forward spatial pressure field (Pa) in the plane* $z = 0.1$ *m at* $t = 2.9$ *ms computed from the spatial pressure field in the plane* $z = 0.05$ *m after enlargement of the array (modulus).* $S_1 = 0.9664$; $S_2 = 1.0453$.

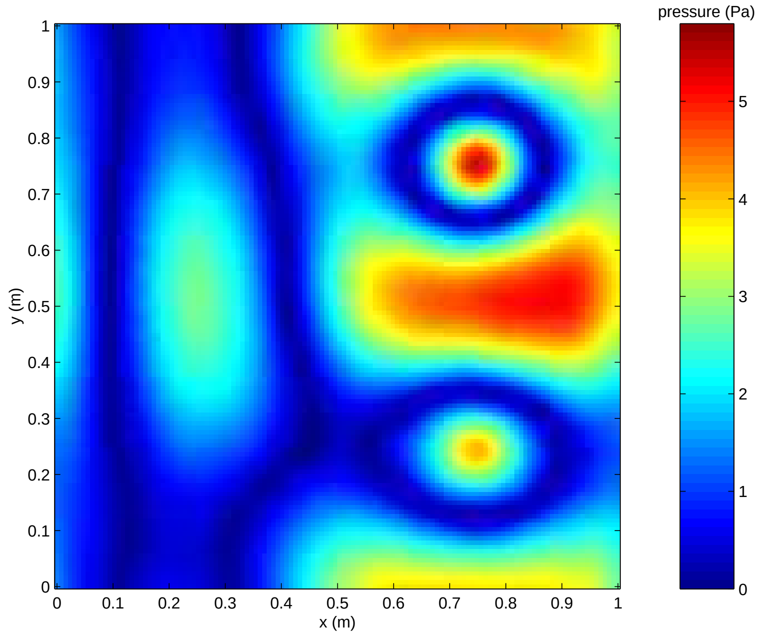

Figure 5: *Forward spatial pressure field (Pa) in the plane* $z = 0.1$ *m at* $t = 2.9$ *ms computed from the spatial pressure field in the plane* $z = 0.05$ *m after oversampling (modulus).* $S_1 = 0.9847$ *;* $S_2 = 1.1575$.

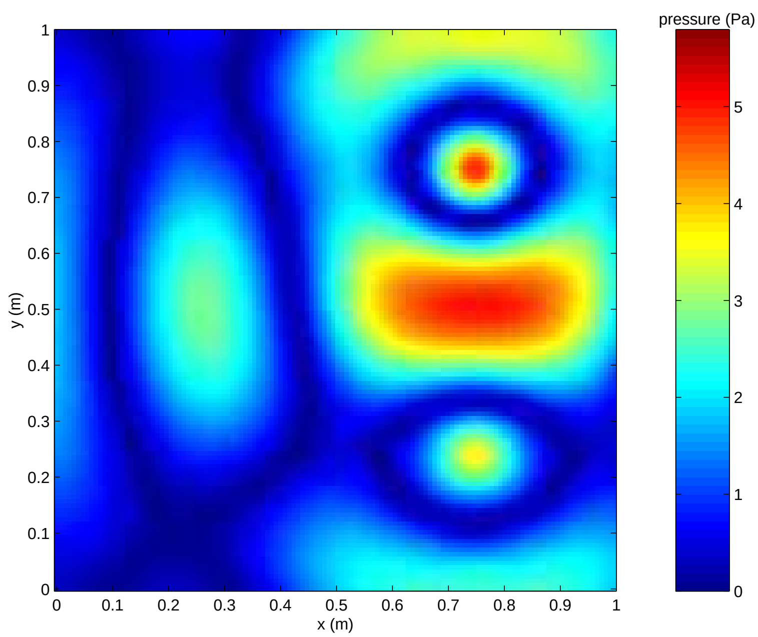

Figure 6: *Forward spatial pressure field (Pa) in the plane* $z = 0.1$ *m at* $t = 2.9$ *ms computed from the spatial pressure field in the plane* $z = 0.05$ *m after oversampling and enlargement of the array (modulus).* $S_1 = 0.9990$ *;* $S_2 = 1.0143$

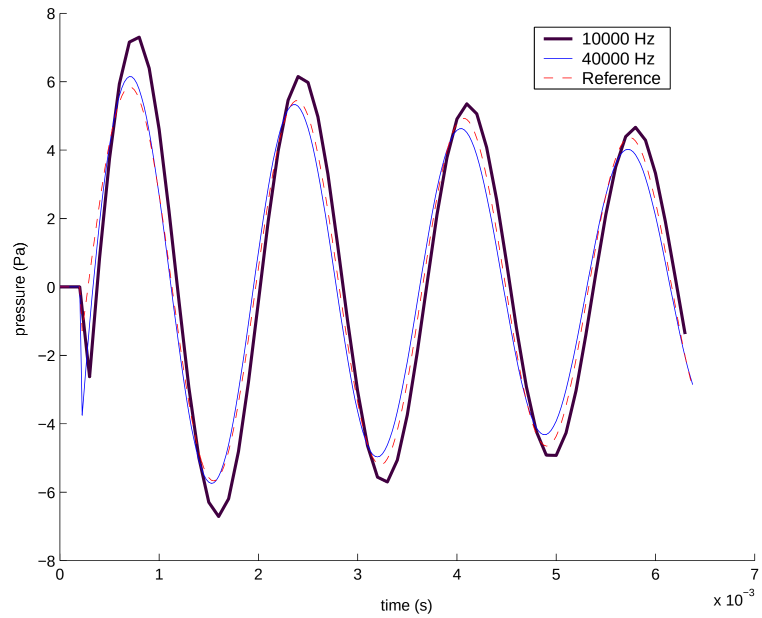

Figure 7: *Time dependant pressure at the position (0.75,0.75,0.1). For* $f_e = 10000$ *Hz,* $T_1 = 0.9773$ *and* $T_2 = 1.1406$*. For* $f_e = 40000$ *Hz,* $T_1 = 0.9928$ *and* $T_2 = 1.0238$*.*

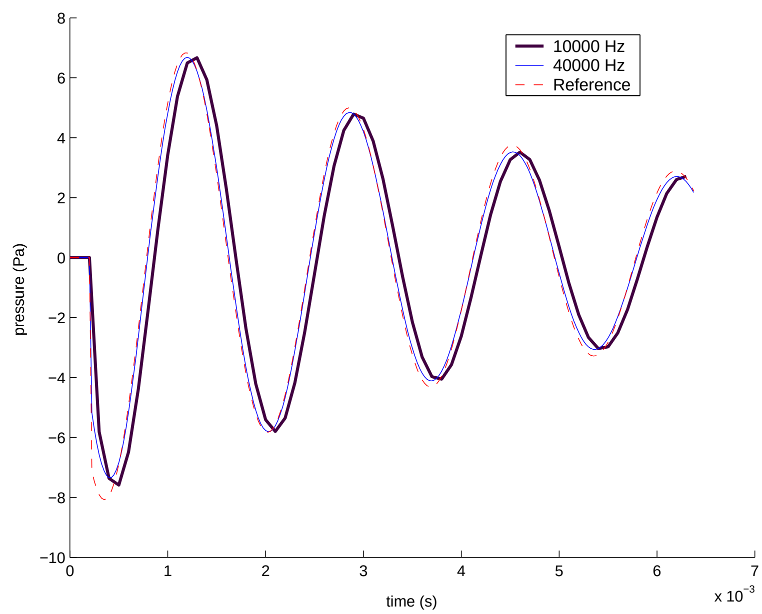

Figure 8: *Time dependant pressure at the position (0.75,0.5,0.1). For* $f_e = 10000$ *Hz,* $T_1 = 0.9582$ *and* $T_2 = 1.0233$*. For* $f_e = 40000$ *Hz,* $T_1 = 0.9977$ *and* $T_2 = 1.0225$*.*