Temporal formulation in the wave number domain of forward propagation of time evolving acoustic pressure fields

Vincent Grulier[1,2], Jean-Hugh Thomas[1], Jean-Claude Pascal[1], Jean-Christophe Le Roux[2]
[1] *Laboratoire d'Acoustique de l'Université du Maine, 72085 Le Mans Cedex 09, France, Email: vincent.grulier@univ-lemans.fr*
[2] *Centre de Transfert de Technologie du Mans, 72000 Le Mans, France, Email: jcleroux@cttm-lemans.com*

Introduction

The wavenumber method combined with the FFT algorithm is established as a faster tool than the Rayleigh integral method for evaluating harmonic acoustic fields from planar radiators whose sound emission is assumed to be stationary [1]. However, this method is not adapted when analysing non-stationary acoustic fields. Therefore this work deals with the reconstruction of non-stationary acoustic fields using a time domain impulse response formulation of the problem in the wavenumber domain.

The theory to reach the expression of the impulse response is described in the first section. The last section contains numerical results concerning the forward propagation of the pressure field resulting from non-stationary sources composed of monopoles. Those results are compared to the direct calculation to validate the formulation.

Theory

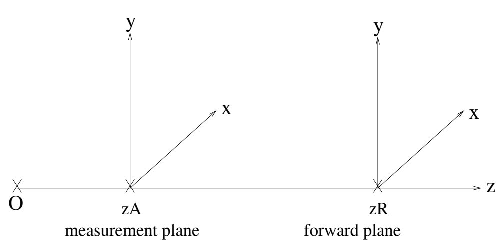

Figure 1: The geometry of interest

Assuming that the sources are only in the region $z < z_A$ the formulation is based on the fact that the time-dependant wavenumber spectrum $P(K_x, K_y, z_R, t)$ in a forward plane $z = z_R$ can be calculated by convolving each component of the time-dependant wavenumber spectrum $P(K_x, K_y, z_A, t)$ in a measurement plane $z = z_A$ ($z_A < z_R$ see Figure 1) with a time domain impulse response h :

$$P(K_x, K_y, z_R, t) = P(K_x, K_y, z_A, t) * h(K_x, K_y, z_R - z_A, t) \tag{1}$$

The expression of the impulse response h is reached by first considering the acoustic propagation described by the wave equation [2] :

$$\Delta p(x, y, z, t) - \frac{1}{c^2}\frac{\partial^2 p(x, y, z, t)}{\partial t^2} = 0 \tag{2}$$

By applying a two-dimensional Fourier transform on x and y to this equation, a differential equation written in a Laplace formalism is obtained. The resolution of this equation gives the expression of the impulse response h where J_1 is the first order Bessel function and $dz = z_R - z_A$ the distance beetween the measurement plane $z = z_A$ and the forward plane $z = z_R$:

$$h(K_x, K_y, dz, t) = \delta\left(t - \frac{dz}{c}\right) - dz\sqrt{K_x^2 + K_y^2}\frac{J_1\left(c\sqrt{K_x^2+K_y^2}\sqrt{t^2 - \frac{dz^2}{c^2}}\right)}{\sqrt{t^2 - \frac{dz^2}{c^2}}}\Gamma\left(t - \frac{dz}{c}\right) \tag{3}$$

Γ is the Heaviside function defined by :

$$\Gamma(t) = \begin{cases} 0 & \text{pour } t < 0 \\ 1 & \text{pour } t \geq 0 \end{cases}$$

The impulse response h can be also written as below with $K_r = \sqrt{K_x^2 + K_y^2}$, $\tau = dz/c$ and $\Omega_r = cK_r$:

$$h(\Omega_r, \tau, t) = \delta(t - \tau) - \tau\Omega_r^2 \frac{J_1\left(\Omega_r\sqrt{t^2 - \tau^2}\right)}{\Omega_r\sqrt{t^2 - \tau^2}}\Gamma(t - \tau) \tag{4}$$

$\tau = dz/c$ is the delay corresponding to the propagation distance dz and Ω_r is the transition pulsation.

In (1) by replacing h with its expression from (3), the time-dependant wavenumber spectrum in the forward plane $P(K_x, K_y, z_R, t)$ is obtained. Then the two-dimensional inverse Fourier transform in space domain yields the instantaneous spatial pressure in the forward plane $p(x, y, z_R, t)$.

Numerical results

The implemented source is composed of two monopoles at the positions $(0.2, 0.2, 0)$ and $(0.6, 0.6, 0)$. Each monopole radiates a signal $s(t) = cos(2\pi ft)e^{-\lambda t}$.The attenuation by an exponential function makes sure the non-stationary condition of the source. The acquisition of the pressure field is simulated by a microphone array located in the measurement plane $z = z_A$ with $z_A = 0.05$ m. At the end of the processing, the resulting pressure field is forward projected from the hologram to the plane $z = z_R$ ($z_R = 0.1$ m). A direct acquisition in the plane

$z = z_R$ of the pressure field resulting from the sources is also computed as a reference to compare to the forward projected pressure field. The reference spatial pressure field in the plane $z = 0.1$ m at $t = 1.9$ ms is plotted on figure 2. Figure 3 shows the spatial pressure field at $t = 1.9$ ms in the plane $z = 0.1$ m obtained by the forward propagation of the spatial pressure field recorded in the plane $z = 0.05$ m. To yield this result, the impulse response h (eq. 3) and the recorded pressure signals are sampled with a sampling frequency $f_e = 10000$ Hz. The microphone array is composed of 17 by 17 microphones equally spaced by 0.0625 m i.e. a 1m by 1m array. The two monopoles are well located in the forward projected spatial pressure field, (cf figure 3) but the amplitude is abnormally superior to the reference amplitude on figure 2 and truncation effects appear lightly.

Two parameters were modified to improve the results : the size of the array and the sampling frequency f_e. The disappearance of the truncation effects with the enlargement of the array was observed. Indeed the measurement plane is assumed to be infinite in extent so to avoid the truncation effects due to the finite size of the hologram in practice, the size of the array is increased. In the same way, the truncation effects increase with the forward distance $z_R - z_A$. On the other hand, the increasing of the sampling frequency f_e results in amplitude closer to the reference. The oversampling of the input signals was used because the impulsional response h is not band-limited in the frequency domain and so it is not possible to choose a priori a value of f_e which satisfies the Shannon theorem. Figure 4 shows the resulting spatial pressure field with $f_e = 40000$ Hz from a 2 m by 2 m array. The coherence beetween the resulting forward projected and the reference spatial pressure fields is good.

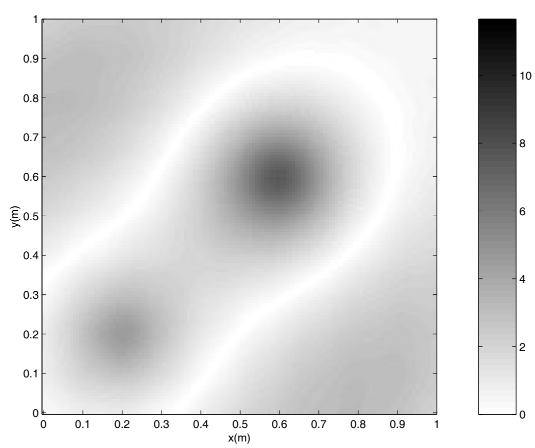

Figure 2: Reference spatial pressure field (Pa) in the plane $z = 0.1$ m at $t = 1.9$ ms

Conclusion

The forward projection of the pressure field radiated by a non-stationary source was presented. The agreement between the results from the direct calculation and those from the forward propagation is fairly good. The rele-

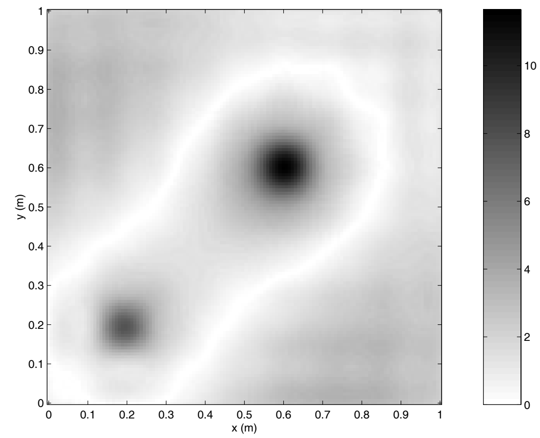

Figure 3: Forward projected spatial pressure field (Pa) at $t = 1.9$ ms in the plane $z = 0.1$ m computed from the spatial pressure field in the plane $z = 0.05$ m

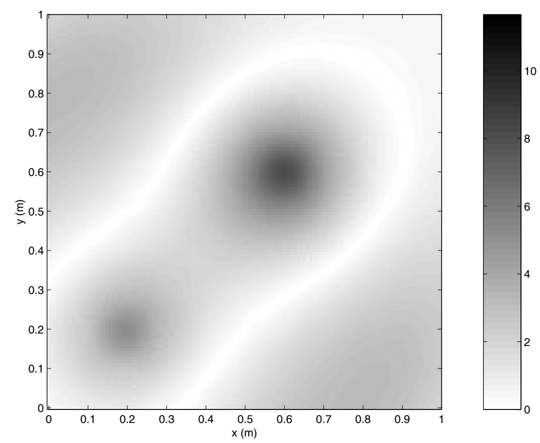

Figure 4: Forward projected spatial pressure field (Pa) at $t = 1.9$ ms in the plane $z = 0.1$ m computed from the spatial pressure field in the plane $z = 0.05$ m after enlargement of the microphone array and oversampling

vance of the method is to provide a real-time prediction of the acoustic wave propagation in the near-field of vibrating structure or in waveguide for applications in active control. The next step of the work would be the synthesis of a numeric filter associated with the impulse response without oversampling and the development of a method based on the inverse of the formulation mentioned above i.e. the backward propagation of time evolving acoustic pressure fields.

References

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