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RADIATION MODES FOR THE IDENTIFICATION OF ACOUSTIC SOURCES

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CONTEXTE

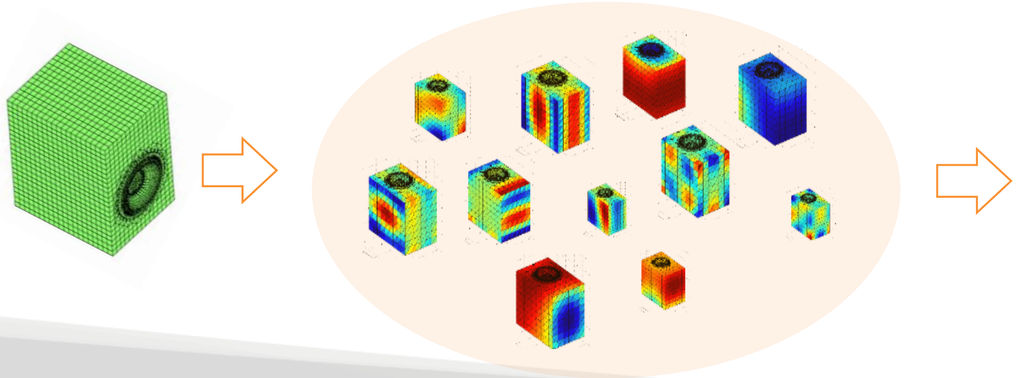
NEED

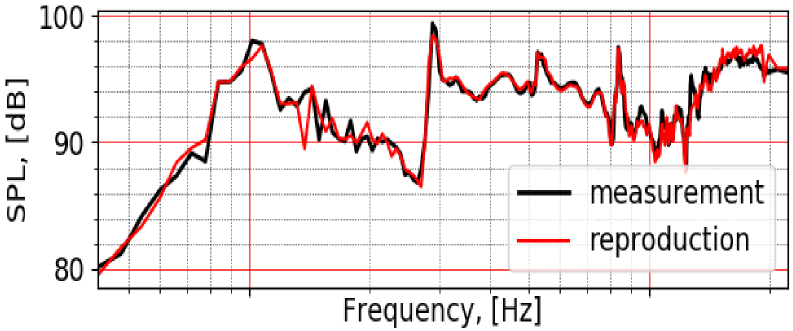
Measurement of the **frequency response** and the **directivity pattern** of a sound system **without expensive facilities**

SOLUTION

Radiation modes (RM)

- ***RM is a base of test functions over the source surface, that represent its radiation***
- ***They do not depend on the vibration pattern, but on geometry of boundary bonditions (BC)***
- ***Set of most efficient ones approximates accurately the radiated sound field***





PRESSURE CALCULATION WITH RM

1. SURFACE ACOUSTIC IMPEDANCE

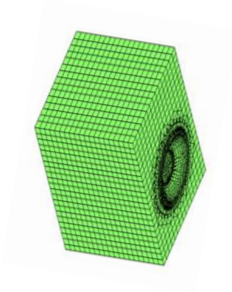
Reaction of the exterior environment on the object vibration

$$\alpha(\boldsymbol{r})p(\boldsymbol{r}) = \int_S \left(\frac{\partial G}{\partial \boldsymbol{n}}(\boldsymbol{r},\boldsymbol{r_s})p_s - G(\boldsymbol{r},\boldsymbol{r_s})\frac{\partial p_s}{\partial \boldsymbol{n}} dS \right), \forall \boldsymbol{r} \in R^3$$

$$\tfrac{1}{2}p_s = \boldsymbol{M}q_s - \boldsymbol{D}p_s$$

$$\boldsymbol{Z} = \boldsymbol{p_s q_s^{-1}}$$

G: Green's function
$\boldsymbol{n}$*: outgoing normal to surface* S
$\boldsymbol{r}$: point outside the surface S
$\boldsymbol{r_n}$: point on the surface S
p_s: pressure on surface S
q_s: surface S volume velocity
$\boldsymbol{Z}$: acoustic impedance



2. RADIATION MODES

$$SVD\{Re(\boldsymbol{Z})\} = \boldsymbol{U}\,\boldsymbol{\Sigma}\,\boldsymbol{V}^*$$

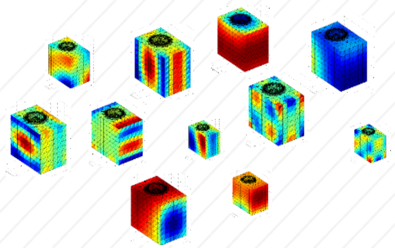
$\boldsymbol{V}$: volume velocity
$\boldsymbol{U}$: pressure
$\boldsymbol{\Sigma}$: relative efficiency of RM

3. PRESSURE ESTIMATION

From a combination of RM that is an approximation of the 3D field (first most efficient ones)

$$p = \boldsymbol{H}w$$

$\boldsymbol{H}$: transfer function between RM and points in space
w: RM weights, that describes the **contribution of each RM**



TARGET PRESSURE ESTIMATION WITH RM

ASSUMING **FOR ANY FREE-FIELD LOCATION**:

$$p = [\boldsymbol{H}]_N w$$

$\boldsymbol{H}$: transfer function between RM and points in space
w: RM weights, that describes the **contribution of each RM**

WEIGHTS CAN BE COMPUTED FROM MEASUREMENTS CLOSE TO THE SOURCE:

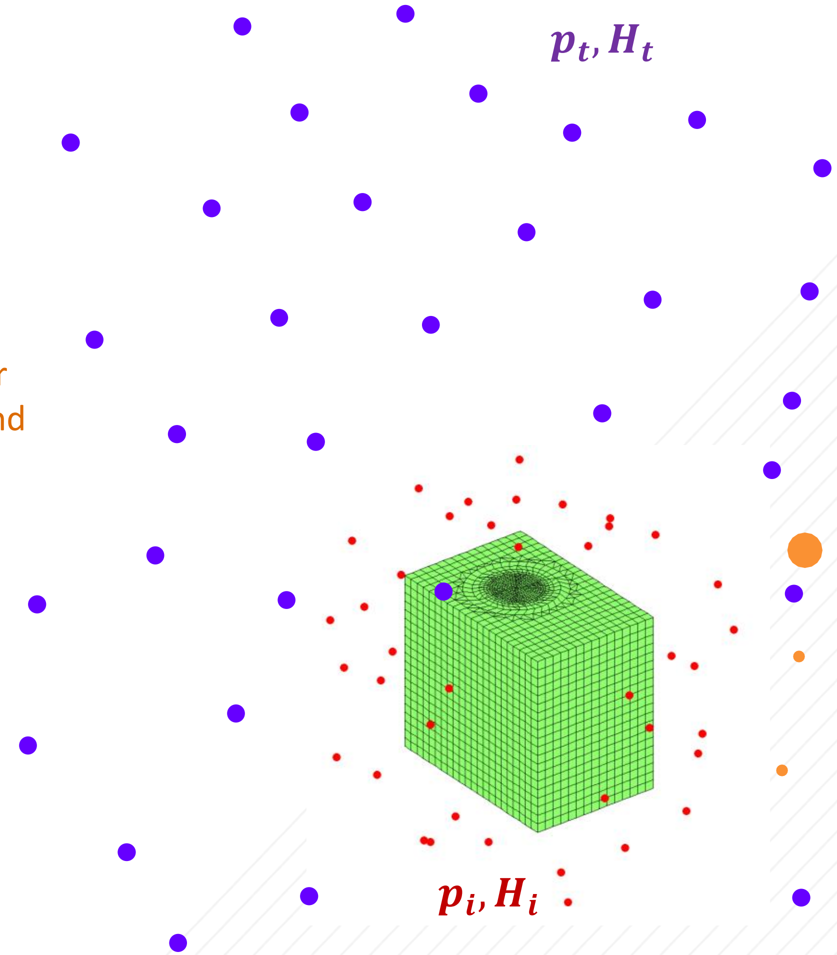
$$w = [\boldsymbol{H}_i^+]_N \, p_i$$

$[\boldsymbol{H}_i^+]_N$: pseudo-inversion of the transfer matrix between **N most efficient RM** and **identification points**

FINALLY, THE **PRESSURE AT TARGET POINTS** IS:

$$p_t = [\boldsymbol{H}_t]_N w$$

$[\boldsymbol{H}_t]_N$ transfer matrix between **N most efficient RM** and **target points**



RM APPROACH RECAP

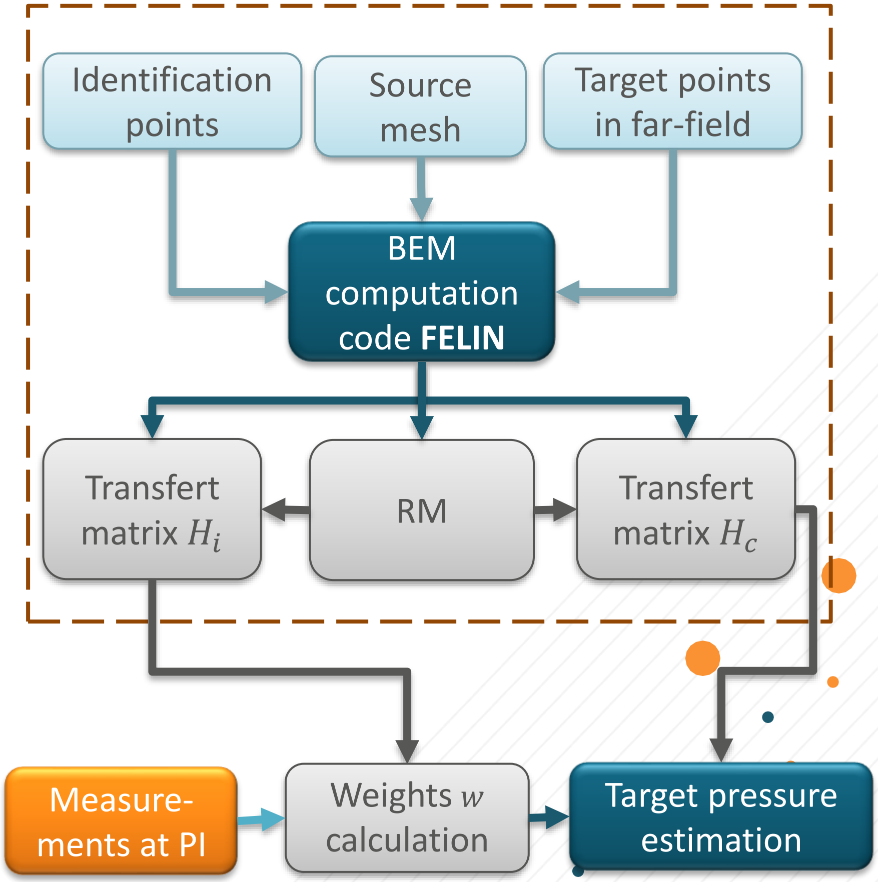
1. DEFINE THE PROBLEM GEOMETRY
   - Source mesh
   - Identification points
   - Target points

2. COMPUTE RM, TRANSFER FUNCTIONS MATRIX & CHOSE THE COMBINATION OF RM (H_i, H_t)

3. MEASURE THE PRESSURE AT IDENTIFICATION POINTS (p_i)

4. COMPUTE RM WEIGHTS (w)

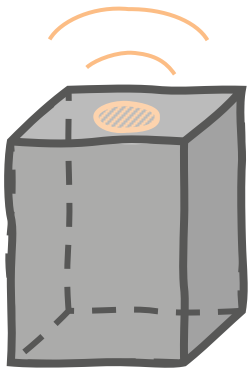
5. ESTIMATE PRESSURE AT OTHER POINTS IN THE FAR FIELD $(p_t = \boldsymbol{H}_t w)$



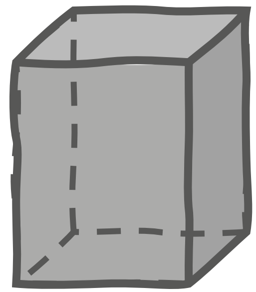
- Reasonable amount of measurements
- Good SNR
- No need in anechoic room

BREAKING THE NEUMANN BC…

- RM ARE COMPUTED USING **NEAUMANN BOUNDARY CONDITION (RIGID WALL)**



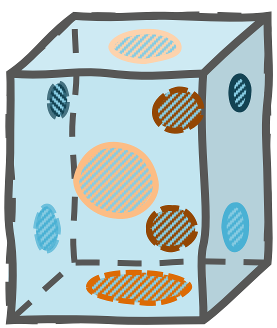
Loudspeaker in a closed box enclosure



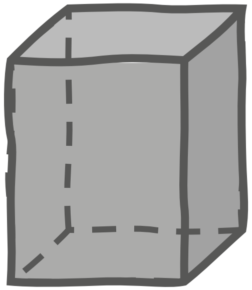
Rigid wall box

$V = 0, \qquad A = 0$

OK



Complex source with openings, large zones with vibrations and admittances



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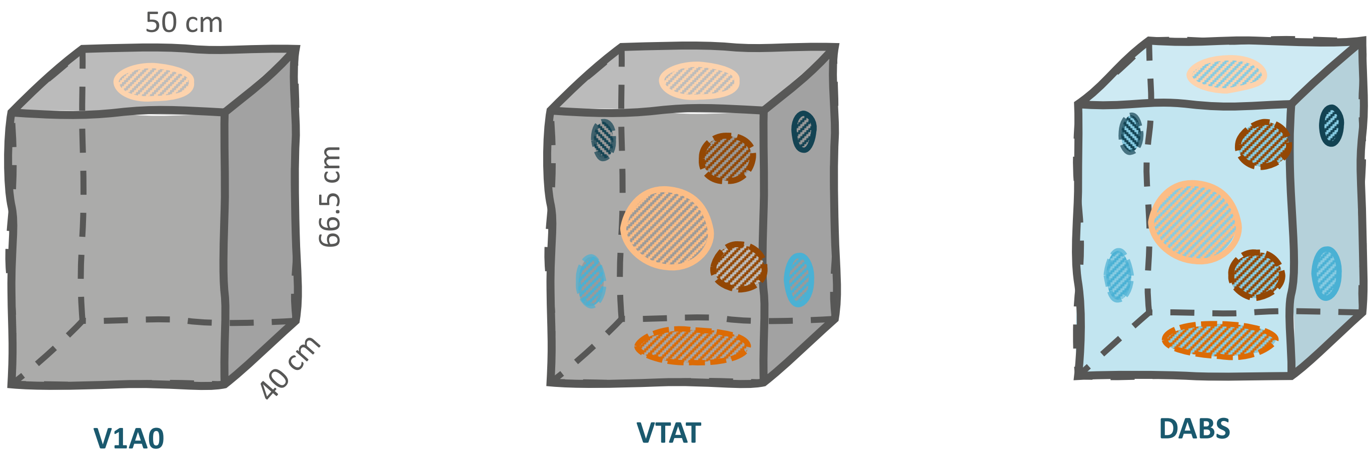
TEST SOURCES



ZONES TYPES:

- **Vibration & admittance**

$f_0 = 44\ Hz$ | $31 - 62\ Hz$

$f_0 = 88\ Hz$ | $62 - 125\ Hz$

$f_0 = 177\ Hz$ | $125 - 250\ Hz$

$f_0 = 707\ Hz$ | $0.5 - 1\ kHz$

- **Admittance only**

$f_0 = 354\ Hz$ | $0.25 - 0.5\ kHz$

$f_0 = 1414\ Hz$ | $1 - 2\ kHz$

$A = 1/\rho c$ | $31\ Hz - 2\ kHz$

- **Rigid wall**

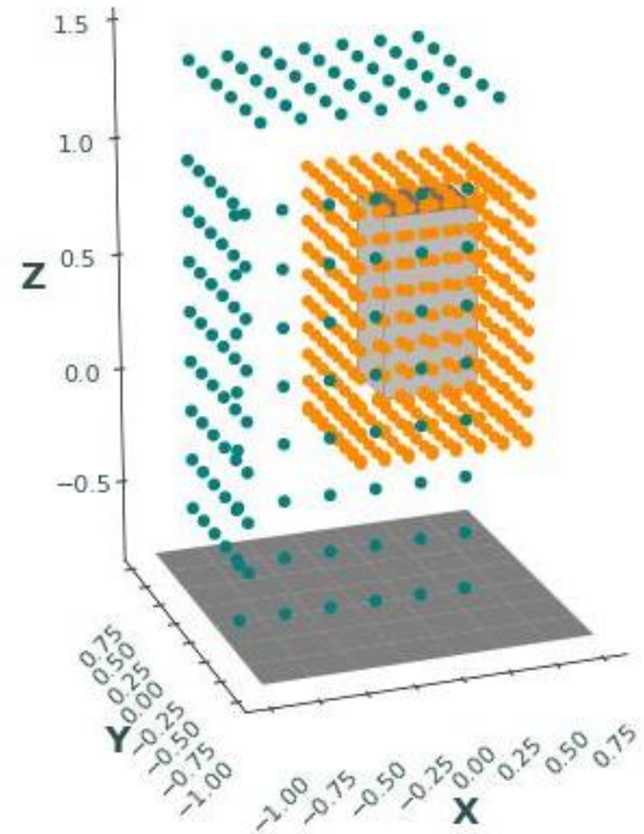
$V = 0, A = 0$

$$\mathrm{V} = \frac{jV_0/Q\ f/f_0}{1 - f^2/f^2 + jf/f_0 Q}$$

$$A = \frac{jA_0/Q\ f/f_0}{1 - f^2/f^2 + jf/f_0 Q}$$

V: Vibration
A : admittance
f: frequency
f_0: central frequency
$V_0 = 1\ m/s$ at f_0
$A_0 = 1/\rho c$ m2s/kg at f_0
$Q = 8$: quality factor

PROBLEM GEOMETRY



PROBLEM GEOMETRY

- **40 x 50 x 66.5 cm3** enclosure
- Reflecting plan at **-0.8m**
- **48, 42 & 48** estimation points ***Pe*** at **~1 m** from surface
- **428** identification points ***Pi*** at **20 cm** from surface

RM COMPUTATION

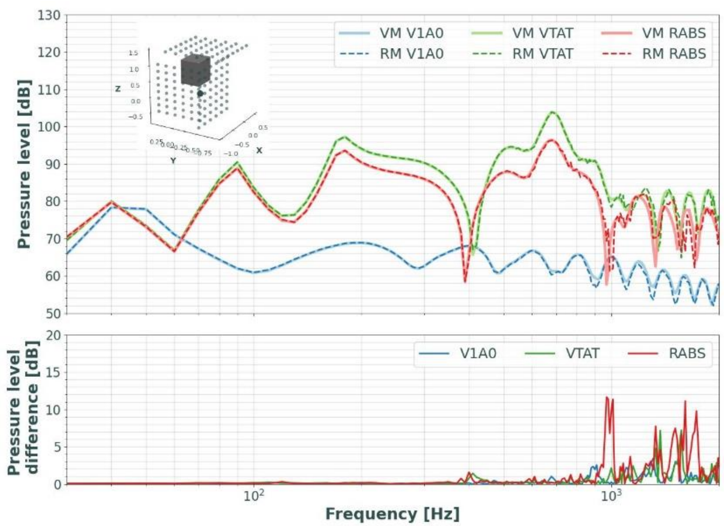
- **Nrm = 90% total efficiency + 20 RM**
- **28 mm** element size mesh (**fmax ≈ 2 kHz for 6 elmts/λ**)
- **Rigid surface assumption** (0 vibation & 0 admittance)
- RM and TF computed with **FELIN**

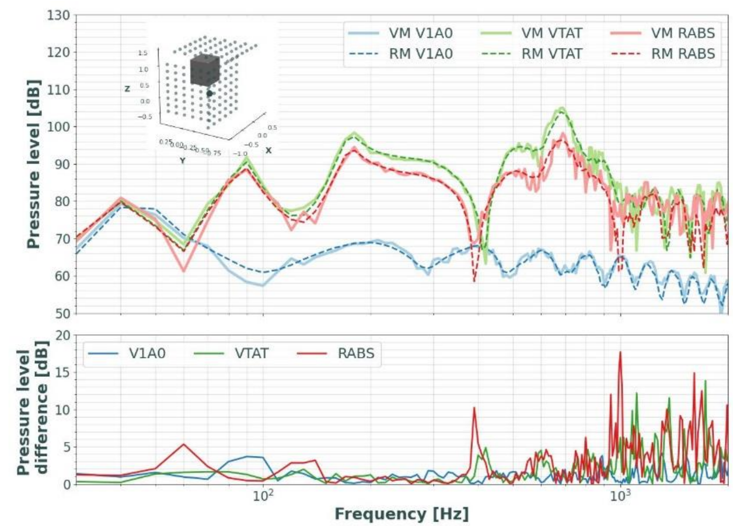
REFERENCE PRESSURE COMPUTATION – VIRTUAL MEASUREMENTS (VM)

- **17 mm** element size mesh (**fmax ≈ 3 kHz for 6 elmts/λ**)
- Vibration configurations: **V1A0, VTAT, DABS**
- Pressures computed with **SYSNOISE**

SIMULATION RESULTS

PRESSURE RESPONSE RECONSTRUCTION | **POINT (-0.9 -0.505 0.37) m**





$P_{Pi} = P_{Pi} + Noise$ → **±1 dB deviation at PI**

$P_{Pi} = P_{Pi} + Noise$ → **±1 dB deviation at Pi**

$P_{Pt} = P_{Pt} + Noise$ → **±2 dB deviation at Pt**

Noise amplitude: $mean(Pi_{RMS}) - 20\ dB$ | **Noise phase:** random

CONCLUSION:

- Reconstruction within ±2 dB
- Most efficient RM are favoring the directions with high pressure → discrepancy at pressure dips

SIMULATION RESULTS

TEMPERATURE EFFECT ON PRESSURE RECONSTRUCTION | **POINT (-0.9 -0.505 0.37) m**

TARGET:

To reconstruct pressure at **20°C**

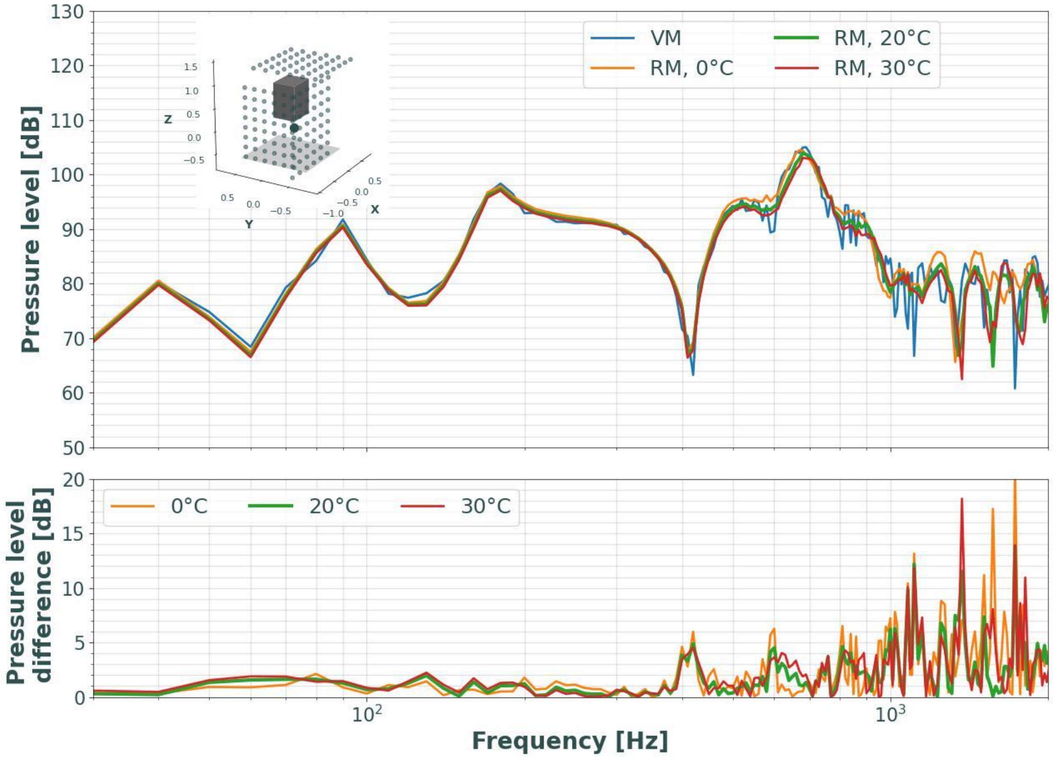
WITH:

- **RM** computed for **+20°C**
- **Identification points** computed for 0°C, **+20°C** and **+30°C**

CONCLUSION:

- Temperature variations have no major effect on the reconstruction

Test case: VTAT



$P_{Pi} = P_{Pi} + Noise$ → **±1 dB deviation at Pi**

$P_{Pt} = P_{Pt} + Noise$ → **±2 dB deviation at Pt**

SIMULATION RESULTS

MESH ELEMENTS SIZE EFFECT ON PRESSURE RECONSTRUCTION | POINT (-0.9 -0.505 0.37) m

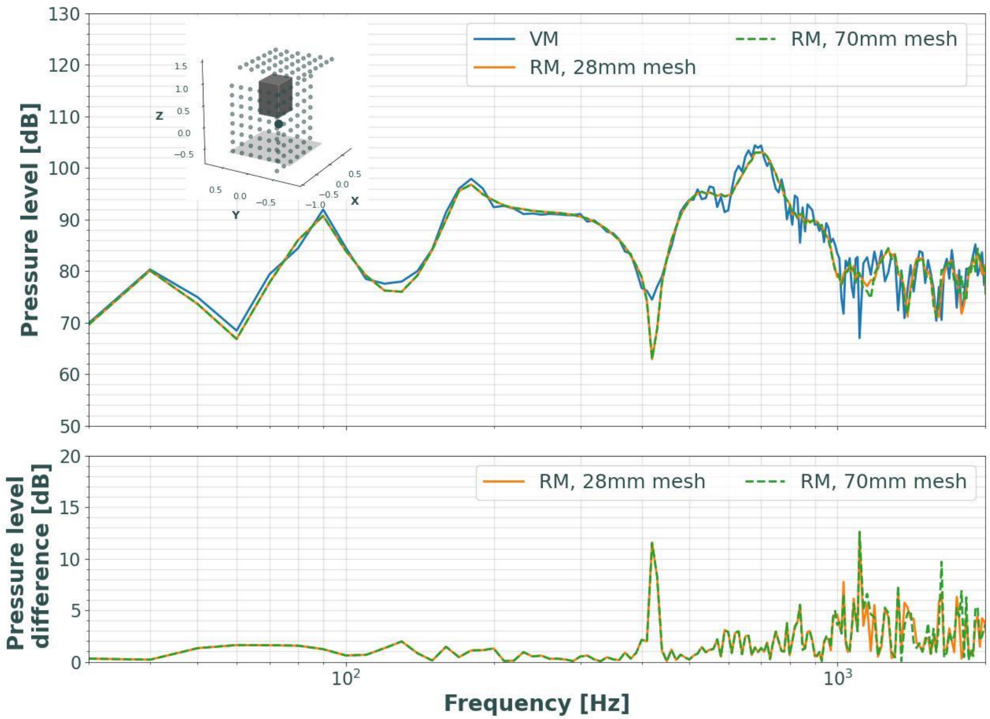
- **28 mm** → fmax ≈ **2 kHz** for 6 elmts/λ
- **70 mm** → fmax ≈ **800 Hz** for 6 elmts/λ

Parameter \ Mesh element length	28mm	70mm
Nb of elements	5620	978
Nb of computed RMs	200	200
Nb of computed frequnecies	158	158
Computation time	54 h	38 min

CONCLUSION:

- Coarser frequency criterion less then 6 elmts/λ

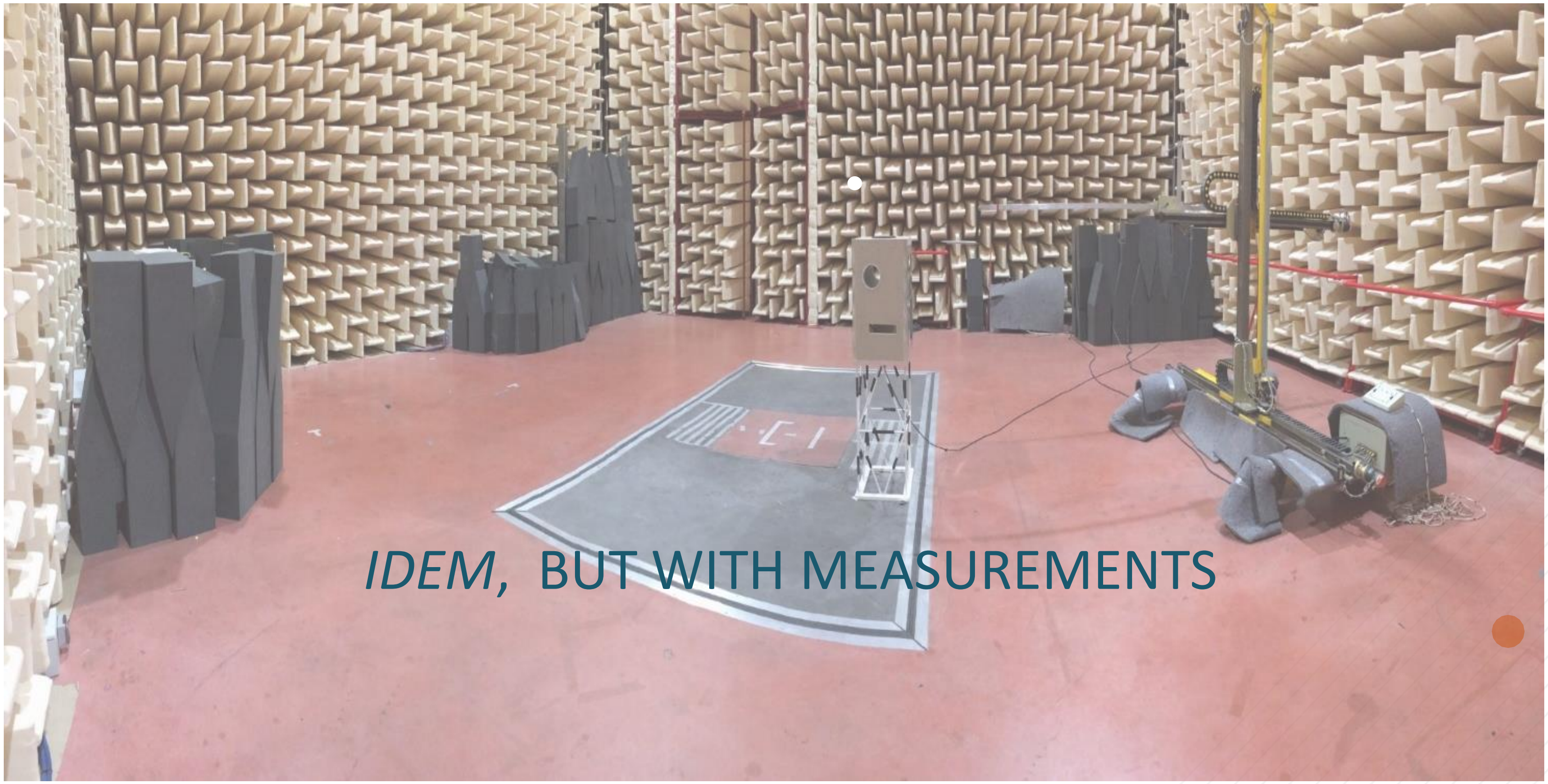
Test case: VTAT



$P_{Pi} = P_{Pi} + Noise$ → **±1 dB deviation at Pi**

$P_{Pt} = P_{Pt} + Noise$ → **±2 dB deviation at Pt**

MEASUREMENT



IDEM, BUT WITH MEASUREMENTS

TEST SOURCES AND RM COMPUTATION

TEST SOURCE 1: Closed box

- 50 x 40 x 66.5 cm
- Scan-speak driver 18S/8531G00 (ø 17 cm)

TEST SOURCE 2: Complex box

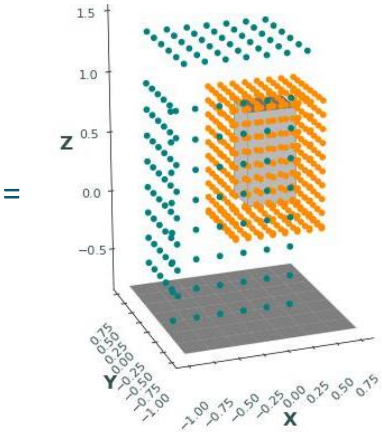
- Traversing pipe (ø12 cm)
- Port (5 x 20 cm)
- Two plexiglas faces (1 mm & 2 mm thickness)
- Parasitic vibrations from metallic supporting structure

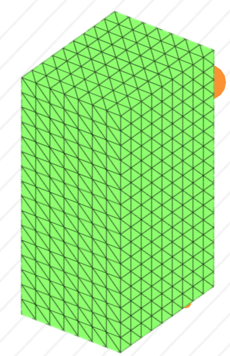
RM COMPUTATION:

- Mesh element size : **70 mm** (Fmax = 800 Hz)
- **Rigid surface**
- Reflecting plan at **-0.8 m**

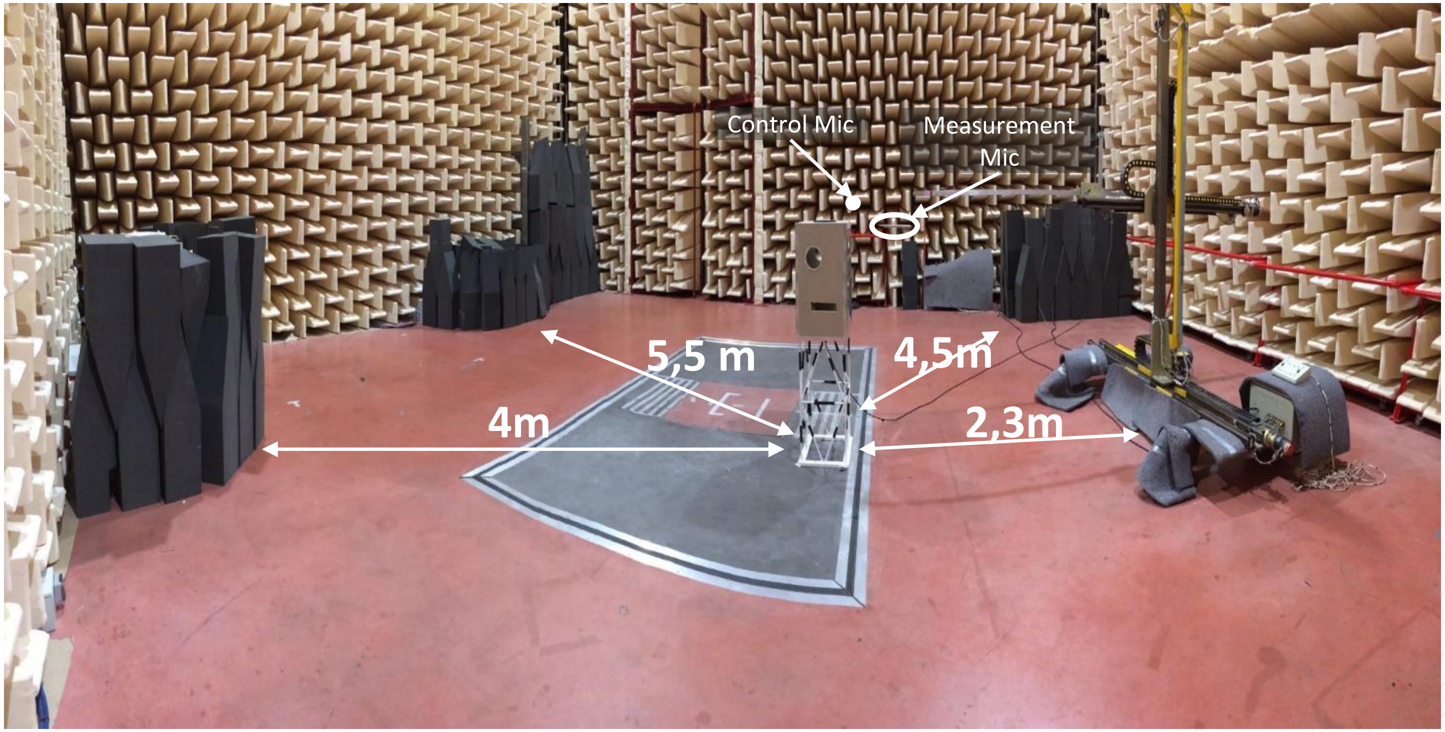
- **428** identification points ***Pi*** at 20 cm from surface
- **48, 42 & 48** estimation points ***Pe*** at **~1 m** from surface
- **200** RM

- **Nrm = 90% total efficiency + 20 RM**





MEASUREMENT SETUP

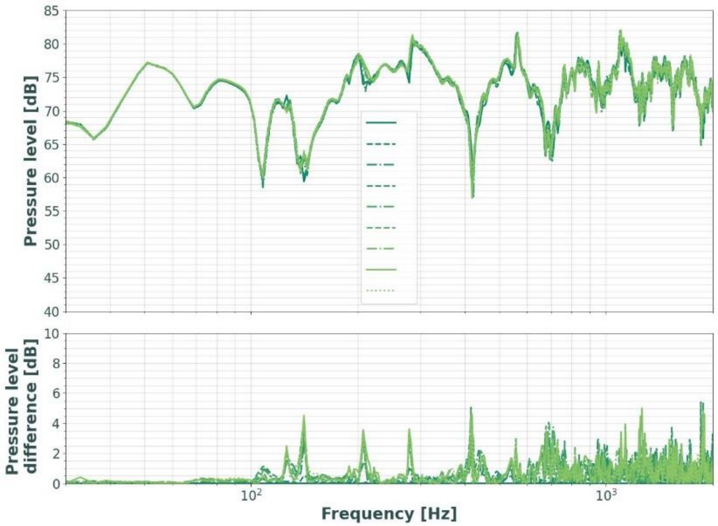


- Measurements during **2 weeks**
- Temperature variations: **+14°C to +17°C**

MEASUREMENT RESULTS

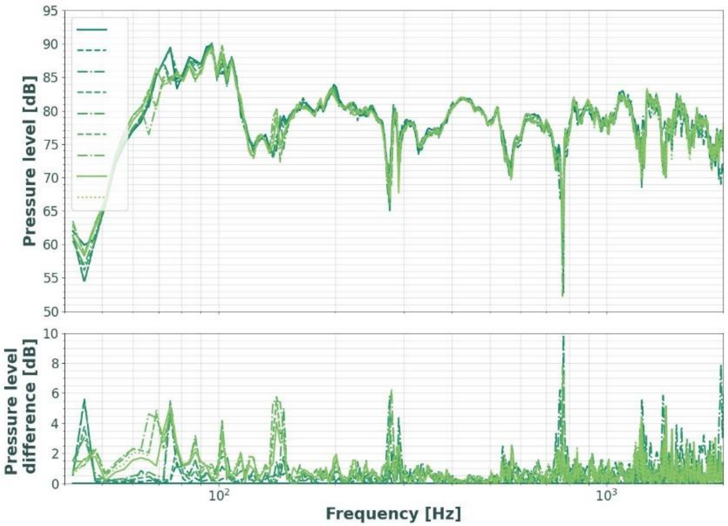
SOURCES **STABILITY MEASUREMENTS**

- 5 meassurements during Pi meas.
- 4 measurements during Pe meas.
- Different position of the robot arm
- On various days
- Different temperatures (+14°C to +17°C)



CLOSED BOX

±2 dB at mid and high frequencies (acceptable)



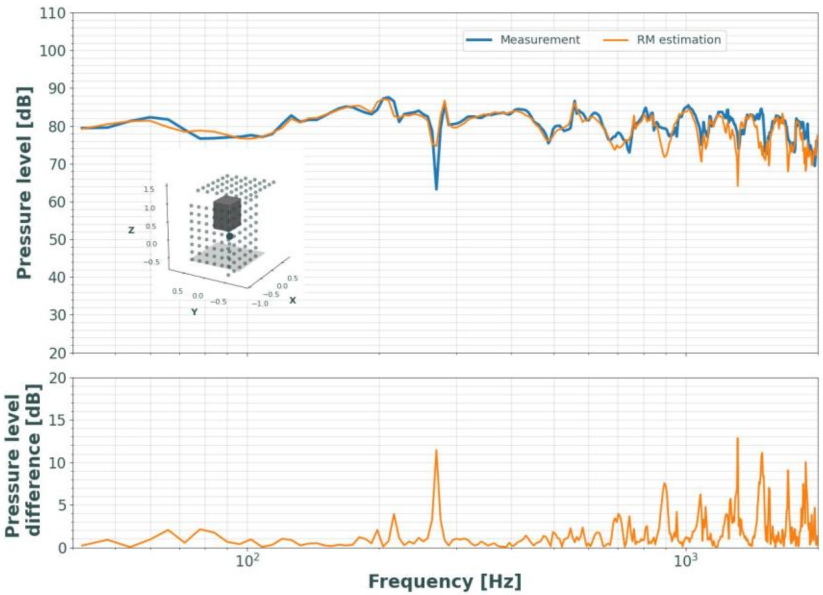
COMPLEX BOX

±3 dB even at low frequencies

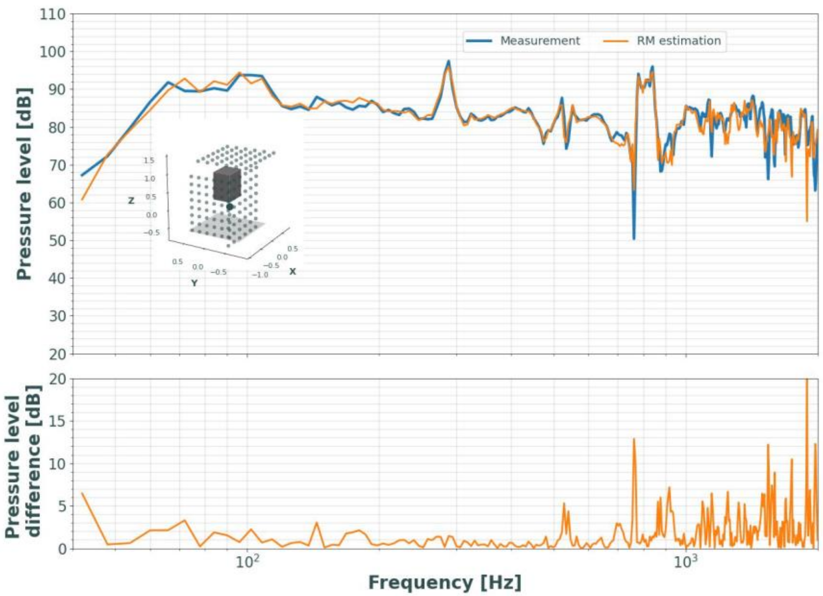
- Behavior of the plexiglas palets
- Evolved during the measurement and probably changed the BC and acoustic radiation

MEASUREMENT RESULTS

PRESSURE RESPONSE RECONSTRUCTION | POINT (-0.9 -0.505 0.37) m



CLOSED BOX



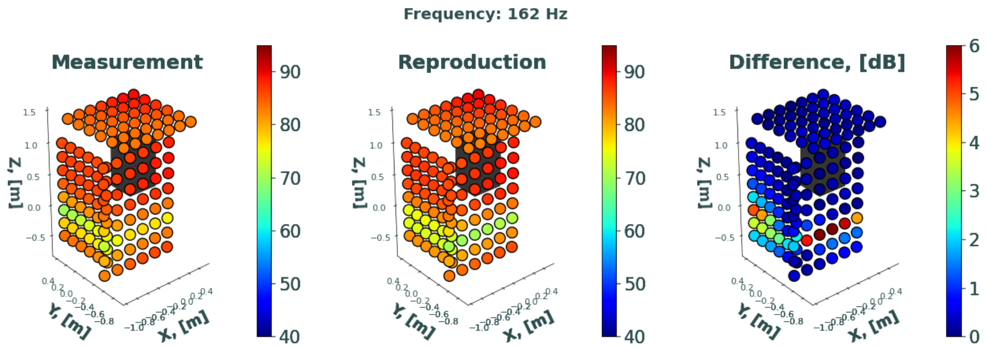
COMPLEX BOX

CONCLUSION:

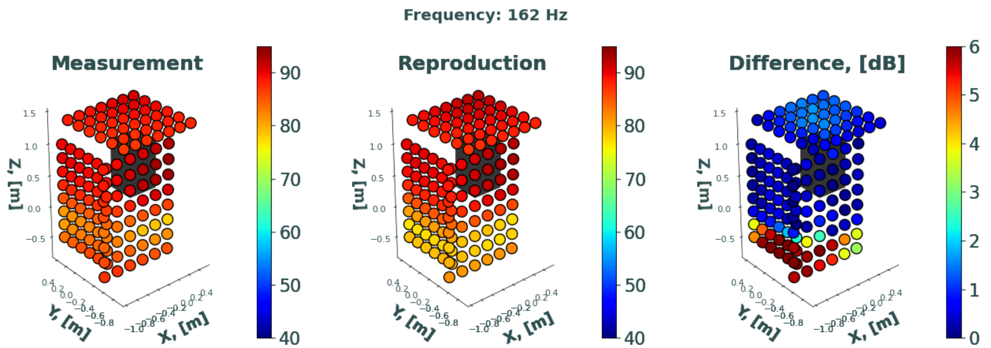
- Reconstruction within ±2 dB
- Discrepancy at pressure dips

MEASUREMENT RESULTS

PRESSURE **CARTOGRAPHY** | 162 HZ



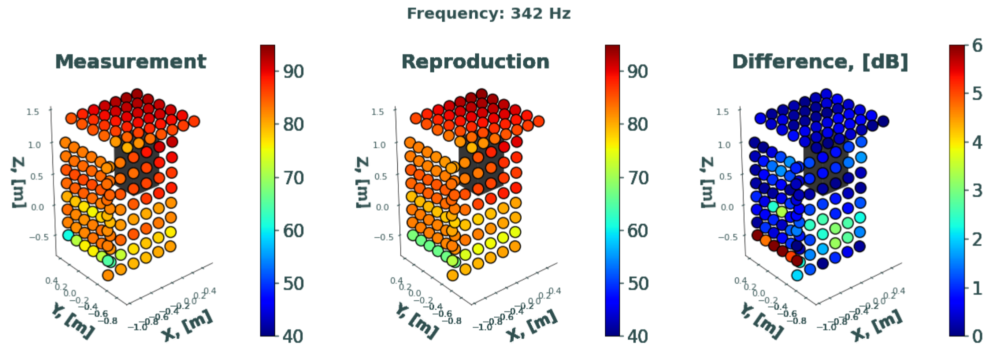
CLOSED BOX



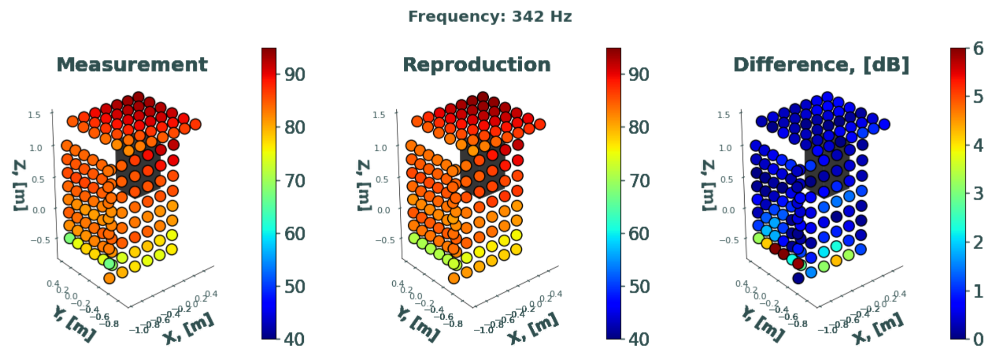
COMPLEX BOX

MEASUREMENT RESULTS

PRESSURE **CARTOGRAPHY** | 342 HZ



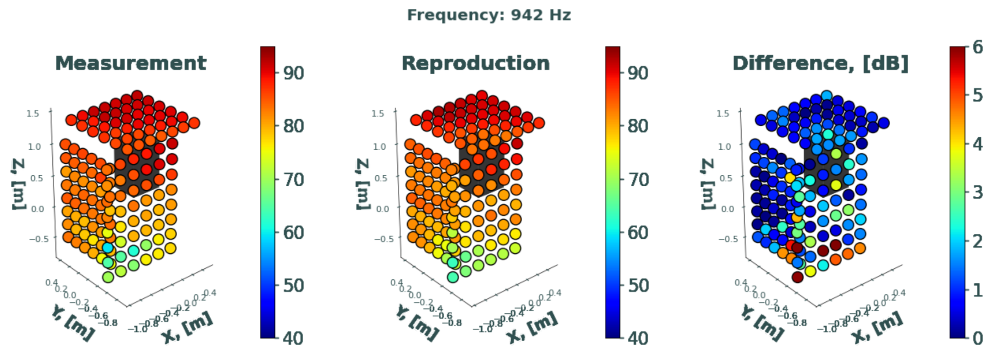
CLOSED BOX



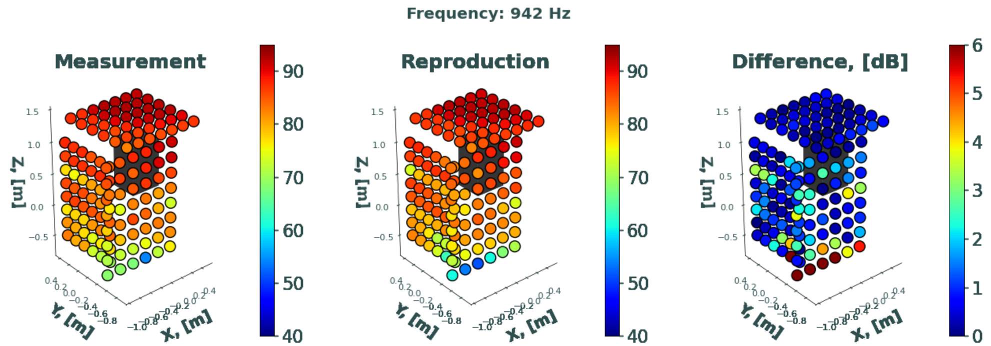
COMPLEX BOX

MEASUREMENT RESULTS

PRESSURE **CARTOGRAPHY** | 942 HZ



CLOSED BOX



COMPLEX BOX

CONCLUSIONS

- **ACCURATE RECONSTRUCTION** EVEN FOR THE **COMPLEX SOURCE**, WITH **SURFACE ADMITTANCE** THAT **DO NOT CORRESPOND** TO THE CALCULATION
- RECONSTRUCTION **DISCREPANCY WITHIN ±2 dB**
- HIGHER DISCREPANCY AT **PRESSURE DIPS**
- POSSIBILITY TO USE **COARSE MESH** FOR THE RM COMPUTATION (≈3 ELMTS/λ)
- ROBUSTNESS TO THE **TEMPERATURE VARIATIONS**

PERSPECTIVES

- INVESTIGATION OF **RM SELECTION** FOR SAVING THE **COMPUTATION TIME** AND BETTER RECONSTRUCTION OF **THE PRESSURE DIPS**
- RM CALCULATED ON A **SURFACE THAT DOES NOT MATCH THE REAL ONE**
- EXTRAPOLATION OF THE **INTENSITY FIELD** FROM PRESSURE MEASUREMENTS
- TEST OF COMPLEX SOURCES WITH **INCOHERENT RADIATION COMPONENTS**

THANK YOU FOR THE ATTANTION !