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Optimisation of anechoic duct termination using line theory

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ABSTRACT

Anechoic terminations are often required for in-duct measurement. Various solutions have been proposed but rarely an optimisation process is proposed. In the present paper we show how to design and optimize an anechoic termination with perforated holes covered by a tissue of known acoustic resistance. From line theory some rules allowing the optimisation are derived. Experiments show that terminations performance can be predicted with a good accuracy leading to efficient solutions with slim dimensions.

1. Introduction

Acoustic characterization of elements - such as exhausts, filters and flexible pipes - is usually performed on dedicated test benches for transfer matrix or radiated noise measurements. The reliabilities of these measurements are generally improved by employing anechoic terminations at the ends of the test rig. In the absence of flow, traditionally, absorbing materials with the shape of a cone are placed at the end of the duct. This is most often done empirically and the anechoicity is not always controlled. Another solution is the use of calibrated acoustic resistance at the end of the duct as proposed in [1]. An example of application is given in [2]. A solution to be mentioned is also the uses of a long tube [3]. In the presence of flow, some kinds of horns have been designed, leading to efficient but bulky solutions [4,5]. Otherwise a perforated duct covered by a tissue is used. To date, little studies have been performed on the subject and while a standard [6] exists, that governs the conception of some terminations, the solutions are not optimized. Thus, the anechoic terminations are usually empirically designed and, at low frequencies, the reflection coefficient remains often too high.

In the present paper we focus on the design of an anechoic termination made with a perforated tube covered by a metallic tissue, i.e. a wiremesh, of calibrated resistivity. We first consider the example of a long tube and that of a tube with a slot of constant width covered by a resistive tissue. These two cases are modelled with the transmission line theory which allows us to derive two simple rules allowing the optimization of the termination. Then, it is shown that the solution can be shortened by using a slot of variable width. Finally, it is shown that the slot can be advantageously be replaced by perforated holes leading to the same result at low frequencies. In order to validate our approach, a termination is realised and measured showing that the reflection coefficient of such a termination can be accurately calculated.

2. Basis

A uniform line can be described by its local characteristics, the series impedance per unit length $\overline{Z}$ and the parallel admittance per unit length $\overline{Y}$. From these quantities the characteristic impedance Z_c and the propagation constant Γ are simply derived:

$$Z_c = \sqrt{\overline{Z}/\overline{Y}} \text{ and } \Gamma = \sqrt{\overline{Z}\overline{Y}}. \tag{1}$$

When such a line of length L is placed at the end of a duct which characteristic impedance is $Z_{c0} \approx \rho c/S$ (ρ the air density, c the speed of sound in air and S the tube cross section), waves coming from the main line are partly reflected at the discontinuity and partly propagate in the line, at the end of which it is reflected (see Fig. 1). So, it can be shown that global reflection coefficient is given by:

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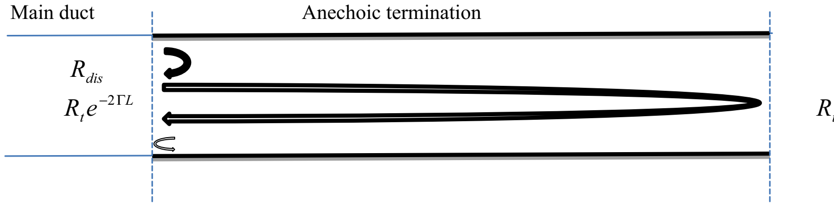

Fig. 1. Scheme of the reflections at the discontinuity and at the end of the termination.

$$R = \frac{R_{dis} + R_t e^{-2\Gamma L}}{1 + R_{dis} R_t e^{-2\Gamma L}} \quad (2)$$

with $R_t = \frac{Z_t - Z_c}{Z_t + Z_c}$ the reflection coefficient at the end of the line (Z_t the impedance at the end of the termination) and $R_{dis} = \frac{Z_c - Z_{c0}}{Z_c + Z_{c0}}$ the reflection coefficient due to the change of characteristic impedance at the junction.

In order to realize an anechoic termination, the objective is to obtain the lowest possible reflection coefficient, or, at least, lower than a given value ε, i.e. $|R| < \varepsilon$. So two conditions are required:

- first, the impedance discontinuity at the junction shall be reduced:

$$|R_{dis}| < \varepsilon_{dis} \quad (3)$$

- second, the damping in the line $\alpha L = \mathrm{Re}(\Gamma L)$, shall be large enough:

$$|R_t| e^{-2\alpha L} < \varepsilon_\alpha. \quad (4)$$

As $|R_t| \leqslant 1$,

$$\alpha L > -\ln(\varepsilon_\alpha)/2 \quad (5)$$

is a sufficient condition for αL.

Globally,

$$\varepsilon_{dis} + \varepsilon_\alpha < \varepsilon \quad (6)$$

is a sufficient condition.

3. Long tube as an anechoic termination

A long pipe is a simple and efficient solution to realize an anechoic termination. For such a pipe the viscothermal losses are sufficient to cancel the wave reflected at the end. In that case, there is no impedance discontinuity since the long pipe has the same diameter as the main duct. The absorption coefficient α is accurately approximated by

$$\alpha = 3.0 \cdot 10^{-5} \sqrt{f}/r, \quad (7)$$

where f is the frequency in Hz and r the radius of the tube in meter. So it is easy to derive from Eq. (5) the minimum length of the tube for a given performance ε of the termination. To fix ideas, considering a tube of 35 mm diameter, to obtain a reflection coefficient lower than $\varepsilon = 0.2$ at 100 Hz, the length shall be 47 m. In practice, such a tube coiled does not take so much place and can be considered as a valuable solution. This is commonly used in capillary tubes. However, in presence of a significant mean flow, pressure losses become too important and another solution have to be found.

4. Tube with a slot covered by a resistive tissue

We consider here a tube with a slot of constant width covered by a resistive tissue of known acoustic resistivity *Res* (homogeneous to ρc) as shown in Fig. 2.

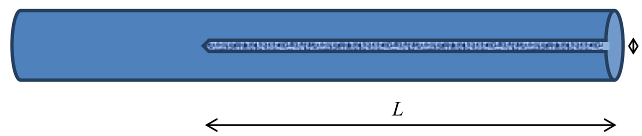

Fig. 2. Scheme of the termination made with a slot covered by a resistive tissue.

In that case, ignoring viscothermal losses in the tube, the series impedance per unit length $\overline{Z}$ and the parallel admittance per unit length $\overline{Y}$ are given by:

$$\overline{Z} = jkZ_{c0} \text{ and } \overline{Y} = jk/Z_{c0} + \overline{G} \quad (8)$$

with $k = \omega/c$, $Z_{c0} = \rho c/S$ the characteristic impedance of the tube of cross section S without the slot and $\overline{G} = e/Res$ the conductance per unit length of the tissue on the slot of width e.

So the characteristic impedance Z_c and the propagation constant Γ are:

$$Z_c = Z_{c0}/\sqrt{1 + \widetilde{G}/jk} \text{ and } \Gamma = jk\sqrt{1 + \widetilde{G}/jk} \quad (9)$$

with $\widetilde{G} = \overline{G} Z_{c0}$.

Considering $\widetilde{G}/k \ll 1$, it comes at first order:

$$\alpha = \mathrm{Re}(\Gamma) \approx \widetilde{G}/2 \text{ and } |R_{dis}| = \left|\frac{Z_c - Z_{c0}}{Z_c + Z_{c0}}\right| \approx \widetilde{G}/4k. \quad (10)$$

It is important to notice that α tends to be constant which implies that the reflection coefficient will not tend to zero with frequency (unless viscothermal losses and radiation are considered).

To fix ideas, let set $\varepsilon_{dis} = \varepsilon_\alpha = 0.1$ at 100 Hz ($k = 1.85$ in standard conditions). This implies $\widetilde{G} = 0.74\ \mathrm{m}^{-1}$ and $L > \ln(\varepsilon_\alpha)/2\alpha \approx$ 3 m which is still rather long.

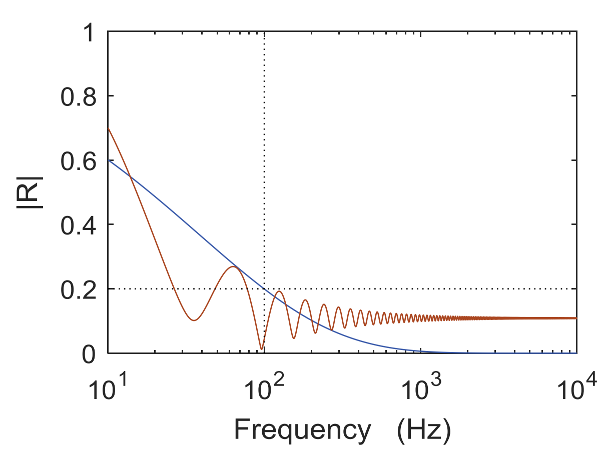

Fig. 3. Modulus of the reflection coefficient. Blue line: tube of 47 m (diameter 35 mm). Red line: tube with a 3 m long slot of constant width covered by a resistive tissue ($\widetilde{G} = 0.74\ \mathrm{m}^{-1}$, viscothermal losses in the tube are ignored). Dotted line indicate the target: $|R| < 0.2$ for $f > 100$ Hz. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

In Fig. 3, the modulus of the reflection coefficient for this solution is compared to that of a 47 m long tube, both solutions fitting into the target.

5. Tube with a slot with variable width covered by a resistive tissue

To shorten the termination, an idea, often used in practice, is to use a slot with an increasing width along the termination [6]. It can be assumed that, for a same amount of losses $\int_0^L \alpha(x)\,dx$, performances will be similar. Simulations show that this is efficient but that, in practice, the reduction factor is limited to 3 or 4. Simulations also show that the function defining the slot width has a low influence, the main parameter being the width at the end (the resistivity of the tissue being given). Results are generally similar to that of the uniform line, but the cut-off is more abrupt and slightly larger than that of the uniform line.

In Fig. 4, an example is given: a termination of length $L = 1$ m with a linear increase of the slot width and the same amount of losses than that of the solution of the previous section ($\widetilde{G}_0 = 0.74\ \text{m}^{-1}$ and $L_0 = 3$ m) i.e. $\int_0^L \alpha(x)\,dx = \alpha_0 L_0$ with $\alpha_0 \approx \widetilde{G}_0/2$ (Eq. (10)).

6. Tube with regularly spaced holes covered by a resistive tissue

In practice the making of a slot on a tube is not straightforward. It is much simpler to drill holes. Then the holes network can be seen as a discretization of the slot and

$$\overline{G} = s/(Res \cdot d) \tag{11}$$

with s the cross section of the hole and d the distance between two holes.

The calculation is based on the transfer matrix method (TMM, see Ref. [7]). The transfer matrix $[Cell]_i$ of a two-port (a piece of tube) makes the link between pressure and volume velocity at the input and the same quantities at the output. So, when N two-ports are connected together, the resulting transfer matrix is the product of the two-ports matrices:

$$[M] = \Pi_{i=1}^{N} [Cell]_i. \tag{12}$$

In the present case a cell is a piece of tube of length d with a hole of cross section s covered by a resistive tissue of nominal acoustic resistance Res. Its transfer matrix is the product of the matrix of the hole and that of the piece of tube. The first is given by:

$$\begin{bmatrix} 1 & 0 \\ s/Res & 1 \end{bmatrix} \tag{13}$$

if only the resistance of the tissue is considered. The second is given by:

$$\begin{bmatrix} \cos(kd) & jZ_{c0}\sin(kd) \\ \frac{j}{Z_{c0}}\sin(kd) & \cos(kd) \end{bmatrix} \tag{14}$$

where k is the wave number (complex if viscothermal losses are considered) and $Z_{c0} \approx \rho c/S$. The transfer matrix of the termination being calculated and assuming a total reflection at the end of the open termination, the input impedance of the termination is $Z_e = C/D$ from which the reflection coefficient $R = \frac{Z_e - Z_{c0}}{Z_e + Z_{c0}}$ can be deduced.

In Fig. 5 a discretization of the solution with a constant slot width is presented. The slot is replaced by 32 holes regularly spaced. Viscothermal losses are taken into account, according to Eq. (7), but not the radiation. It appears that the slot and the holes lattice lead to similar results as long as the half wave length is larger than the distance between two holes. When the distance between holes is a multiple of the half wavelength, holes are no more efficient which result in high values of the reflection coefficient. If necessary, this can be avoided by reducing the distance between the holes or by drilling the holes unregularly. In practice the anechoic termination is used generally up to the cut-off frequency of the tube. So it is sufficient to set the distance between the holes to a little bit less than a tube diameter.

7. Optimisation procedure

To resume, the maximum acceptable reflection coefficient ε being given in a frequency band $[f_{min}\, f_{max}]$ the optimization procedure is the following:

i. The condition $d < c/(2\, f_{max})$ gives the minimum distance d between two holes.

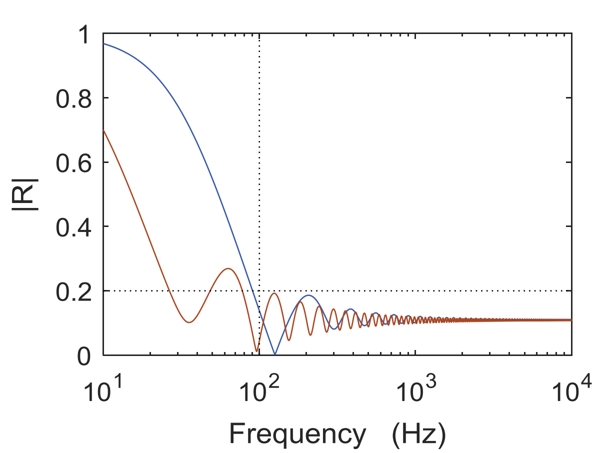

Fig. 4. Modulus of the reflection coefficient. Red line: tube with a 3 m long slot of constant width covered by a resistive tissue ($\widetilde{G} = 0.74\ \text{m}^{-1}$, viscothermal losses in the tube are not taken into account). Blue line: tube with a 1 m long slot of variable width covered by a resistive tissue ($\widetilde{G}$ varying from its initial value $\widetilde{G}_0 = 0.74\ \text{m}^{-1}$ to $5\widetilde{G}_0$, viscothermal losses in the tube are not taken into account). Dotted line indicate the target: $|R| < 0.2$ for $f > 100$ Hz. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

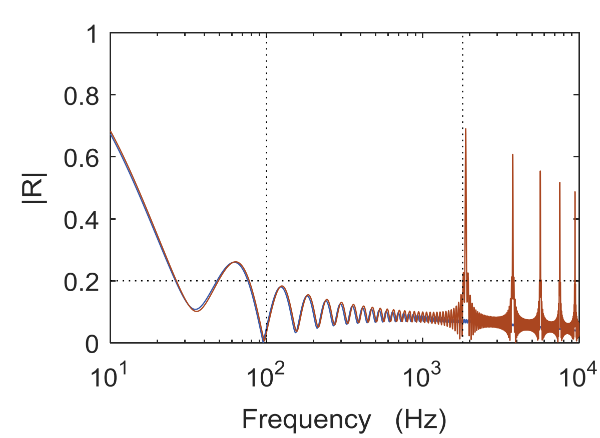

Fig. 5. Modulus of the reflection coefficient. Blue line: uniform slot as in Fig. 3. Red line: 32 holes regularly spaced on same length as the slot. Dotted line indicate the target: $|R| < 0.2$ for $100 < f < 1800$ Hz. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

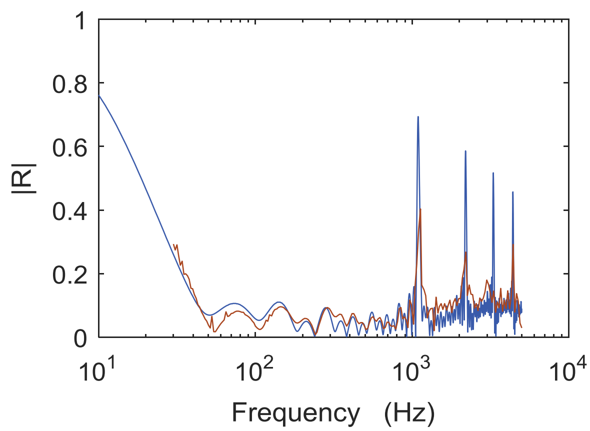

Fig. 6. Modulus of the reflection coefficient of an anechoic termination with regularly spaced holes covered by a resistive tissue. Blue: theory; Red: measurement. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

ii. Setting the reflection coefficient at the input to $\varepsilon_{dis} = \varepsilon/2$ at $f_{\min}$, gives the first hole cross section s_0 (Eqs. (10) and (11)): $\frac{s_0}{S} = 2\varepsilon \frac{Res}{\rho c} k_{\min} d$, the resistance of the tissue *Res*, the cross section of the tube S and $k_{\min} = 2\pi f_{\min}/c$ being given.
iii. Setting the absorption along the termination to $\varepsilon_\alpha = \varepsilon/2$ gives the length of the uniform line (Eqs. (5), (11) and (10)): $L = \ln(\varepsilon/2) \frac{Res}{\rho c} \frac{S}{s_0} d$. The number of holes is $N = [L/d]$.
iv. Reduce the length by increasing the cross section of the holes linearly to a factor of 4 or 5. For example 30 holes can be reduced to 3 holes of cross section s_0, 3 of cross section $2s_0$ 3 of cross section $3s_0$, and 3 of cross section $4s_0$, reducing the length by a factor of 30/12.

All the dimensions being determined, it is good to check the performances of the termination by calculating the reflections coefficient via the TMM.

8. Experimental validation

Since 2012, many terminations have been made at CTTM based on a theoretical optimization [8]. In Fig. 6 is presented a 2.5 m long termination of 35 mm diameter with 16 holes of diameters ranging from 8 mm to 16 mm and covered by a tissue of calibrated resistivity 230 Rayls MKS. The distance between holes is 156 mm so that the limit frequency for which this distance is a half wave length is beyond 1000 Hz. With these dimensions, the reflection coefficient is lower than 10% between 40 and 1000 Hz which correspond to the target. Results show that the predictability of the performances of the termination is very good provided that the resistivity of the tissue is accurately known.

9. Conclusion

It is shown that the optimization of an anechoic termination with perforated holes is a rather easy task, since only two independent criteria have to be considered: a not to large impedance discontinuity and a sufficient damping. Following the steps described in the present paper, high performances can be achieved without leading to cumbersome solutions. Finally, it can be wondered why such an optimization is not usually done. This is probably because the availability of tissue of calibrated resistivity is still not well known.

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