

Inverse acoustic characterization of rigid frame porous materials from impedance tube measurements

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Abstract

This paper presents a method for the inverse characterization of rigid frame porous materials using audible frequency acoustic measurements in an impedance tube. The characterizing parameters are the six acoustical parameters in the Johnson-Lafarge model. The proposed method is based on a minimization process, where the quantities of interest are found as the minimizing values for the difference between measured and modelled density and compressibility. Measurements are performed in an impedance tube and a scattering matrix formulation is used to obtain the reflection and transmission coefficients R and T , found as the elements of the scattering matrix, and to obtain the effective density and compressibility. Five different porous materials are tested and the results of the inversion process are compared to direct measurements of the acoustical quantities as well as to an already established recovery method [Panneton&Olny, JASA 2006]; [Olny&Panneton, JASA 2008]. The results are also validated against measurements with a rigid backing. It is found that the current method can be used to recover the material parameters quickly and reliably.

1. INTRODUCTION

The purpose of this paper is to present a new method of characterizing sound absorbing porous media. The characteristics of a medium can be thought of as the parameters and their values that are present in a model describing the material. If the frame of the material is considered rigid and motionless, the medium can be modeled as an equivalent fluid. Empirical models, such as the Delany-Bazley (1) or Miki (2) ones, are based purely on power laws. Semi-phenomenological models on the other hand include measurable physical parameters in their description of the materials. Good introductions to the different models can be found for example in Refs. (3), (4) and (5).

The method described in this paper uses a Johnson(6)-Champoux-Allard(7)-Lafarge(8) (JCAL) semi-phenomenological model and is based on recovering the parameters from the material's complex density $\tilde{\rho}$ and compressibility \tilde{K} . They are measured in the range between 200 and 6500 Hz. With the proposed

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method it is possible to recover porosity ϕ , tortuosity α_∞ , characteristic viscous and thermal lengths Λ and Λ' , flow resistivity σ and thermal resistivity σ' . Inversion for the parameter recovery is based on fitting analytical curves, that are calculated from the estimated parameters, to measured curves in the least-squares sense. Equations for flow and thermal resistivities σ and σ' are rewritten in a more physical way to increase sensitivity of the inversion.

2. THEORY

Consider an isotropic, homogeneous, open-cell porous medium. Assume the skeleton to be motionless so it can be modeled as an equivalent fluid. Such a medium can support only sound waves and can be characterized by an equivalent dynamic density $\tilde{\rho}_{\text{eq}}$ and a dynamic bulk modulus \tilde{K}_{eq} . This assumption is valid when the fluid-structure coupling is weak and the wavelength is much larger than the characteristic dimensions of the pores (3).

In the rigid frame model sound waves are attenuated due to viscous and thermal losses which are represented by $\tilde{\rho}_{\text{eq}}$ and \tilde{K}_{eq} , respectively. The tilde over certain functions indicates that they are complex and frequency-dependent. Several semi-phenomenological models have been developed for relating the acoustical behavior of the material to its geometrical properties. These include the Johnson *et al.*(6) and Champoux-Allard (7) models for predicting the density, and the Pride *et al.*(9) and Lafarge (8) models for predicting the bulk modulus.

The equivalent density of the porous medium can be written as

$$\tilde{\rho}_{\text{eq}} = \frac{\rho_0}{\phi} \tilde{\alpha}(\omega) , \quad (1)$$

where ρ_0 is the density of the saturating fluid, ϕ the open porosity and $\tilde{\alpha}(\omega)$ the dynamic tortuosity. In a same fashion the equivalent bulk modulus can be written like

$$\tilde{K}_{\text{eq}} = \frac{\gamma P_0}{\phi} \left(\gamma - \frac{\gamma - 1}{\tilde{\alpha}'(\omega)} \right)^{-1} , \quad (2)$$

where P_0 is the static pressure and γ the specific heat ratio. The parameter $\tilde{\alpha}'(\omega)$ has been defined as a homologue to $\tilde{\alpha}(\omega)$, representing the thermal tortuosity (10).

2.1 Johnson et al. model

The Johnson *et al.* model (6) is a semi-phenomenological model used to describe the complex density of the porous medium. In this model, adopting the $e^{-i\omega t}$ convention, the dynamic tortuosity is written as

$$\tilde{\alpha}(\omega) = \alpha_\infty \left[1 + \frac{i\phi\sigma}{\rho_0\alpha_\infty\omega} F(\omega) \right] , \quad (3)$$

where $F(\omega)$ is a shape function defined by

$$F(\omega) = \sqrt{1 - i\omega\eta\rho_0 \left(\frac{2\alpha_\infty}{\phi\sigma\Lambda} \right)^2} , \quad (4)$$

where η is the viscosity of the saturating fluid, α_∞ , σ and Λ are the tortuosity, flow resistivity and viscous characteristic length(11) of the porous medium, respectively.

2.2 Lafarge et al. model

The Lafarge *et al.* model (10) is also a semi-phenomenological model used to describe the dynamic bulk modulus of the porous medium. In this model the thermal tortuosity is written as

$$\tilde{\alpha}'(\omega) = 1 + \frac{i\phi\sigma'}{\rho_0\alpha_\infty\text{Pr}\omega} G(\omega) , \quad (5)$$

where $G(\omega)$ is given by

$$G(\omega) = \sqrt{1 - i\omega\eta\rho_0\text{Pr} \left(\frac{2\alpha_\infty}{\phi\sigma'\Lambda'} \right)^2} , \quad (6)$$

where Λ' is the thermal characteristic length, σ' is the thermal resistivity and Pr is the Prandtl number.

3. RECOVERY METHOD

Using the Johnson and Lafarge models to describe the dynamic functions $\tilde{\rho}_{\text{eq}}$ and \tilde{K}_{eq} of the equivalent fluid, there are six acoustical parameters to estimate, namely $\phi, \alpha_\infty, \Lambda, \Lambda', \sigma$ and σ' . However, by making a substitution of (6, 12)

$$c = \frac{\Lambda^2 \sigma \phi}{8\eta\alpha_\infty}, \quad c' = \frac{(\Lambda')^2 \sigma' \phi}{8\eta\alpha_\infty}, \quad (7)$$

new expressions can be written for $\tilde{\rho}_{\text{eq}}$ and \tilde{K}_{eq} that are dependent on less parameters, namely $\tilde{\rho}_{\text{eq}} = \tilde{\rho}_{\text{eq}}(\phi, \alpha_\infty, \Lambda^2, c)$ and $\tilde{K}_{\text{eq}} = \tilde{K}_{\text{eq}}(\phi, (\Lambda')^2, c')$. This is physically more valid since in reality the compressibility does not depend on tortuosity. Moreover, now there is only one parameter that $\tilde{\rho}_{\text{eq}}$ and \tilde{K}_{eq} both depend on: the porosity ϕ , which will make the minimization process more sensitive to the parameters. Recovery of the thermal resistance σ' is possible because the considered frequency range includes frequencies between the isothermal and adiabatic regime, where the effect of σ' is the largest. The modified dynamic tortuosities read

$$\tilde{\alpha}(\omega) = \alpha_\infty \left[1 + i \frac{8\eta c}{\omega \rho_0 \Lambda^2} \sqrt{1 - i \rho_0 \omega \frac{\Lambda^2}{16\eta c^2}} \right], \quad (8a)$$

$$\tilde{\alpha}'(\omega) = 1 + i \frac{8\eta c'}{\omega \rho_0 (\Lambda')^2 \text{Pr}} \sqrt{1 - i \rho_0 \omega \text{Pr} \frac{(\Lambda')^2}{16\eta (c')^2}}. \quad (8b)$$

The function to be minimized is formed as the L_2 norm of the difference between measurements of $\tilde{\rho}_{\text{eq}}$ and \tilde{K}_{eq} and their analytically constructed counterparts as

$$f(\boldsymbol{\omega}; \boldsymbol{\theta}) = \left\| \tilde{\rho}_{\text{eq}}^{\text{meas}}(\boldsymbol{\omega}) - \tilde{\rho}_{\text{eq}}^{\text{model}}(\boldsymbol{\omega}; \phi, \alpha_\infty, \Lambda^2, c^2) \right\|_2 + \left\| \tilde{K}_{\text{eq}}^{\text{meas}}(\boldsymbol{\omega}) - \tilde{K}_{\text{eq}}^{\text{model}}(\boldsymbol{\omega}; \phi, (\Lambda')^2, (c')^2) \right\|_2, \quad (9)$$

where $\boldsymbol{\theta}$ denotes all the acoustic variables and $\boldsymbol{\omega}$ is a vector containing a discrete set of relevant range of frequencies. The cost function must be formed as a sum of the two functions $\tilde{\rho}_{\text{eq}}$ and \tilde{K}_{eq} because they both contain information of ϕ . That way, when all the other parameters are independent, a good estimate for ϕ will be acquired. One way to solve the minimization problem

$$\hat{\boldsymbol{\theta}} = \arg \min_{\boldsymbol{\theta}} f(\boldsymbol{\omega}; \boldsymbol{\theta}), \quad (10)$$

is to use the Nelder-Mead simplex algorithm (13), which is an iterative method that does not need numerical or analytical gradients. In the recovery process the parameters are also bounded to reasonable values, such as $\phi \in [0, 1]$ and $\alpha_\infty \in [1, 3]$.

4. RESULTS

The present method was applied to five different porous materials, each with a diameter of 30 mm and thickness varying between 16 mm to 40 mm. The materials were soft polyurethane foam, melamine foam, felt, normal glass wool and Isover[®] Calibel glass wool, see Figure 1. The proposed method is compared to direct measurements of the acoustical properties (flow meter (14), ultrasonic measurements(15)) as well as to recovery methods proposed in Refs. (16) and (17). With the direct measurements it was not possible to measure Λ' and σ' , but Λ' can be approximated by multiples of Λ . With a method proposed in Ref. (18) Λ and Λ' could be measured independently, but it is somewhat difficult to implement.

Parameters obtained from the proposed method, as well as those obtained from the reference method and direct measurements are shown in Table 1. Figure 2 presents all the measured curves of R , T , $\tilde{\rho}_{\text{eq}}$ and \tilde{K}_{eq} along with the reconstructed curves based on the parameter values obtained from the inversion process. Real and imaginary parts are presented separately. Finally, the figure also shows an analytically calculated reflection coefficient for a case of porous material backed with a rigid wall, along with the corresponding measurement.

The ripples in the measured curves that are visible especially in the wools around 2000 Hz are caused by Biot waves. They result from motion of the frame, which is not accounted for in the currently used model. However, the overall effect of the Biot waves on the reconstruction is not too large since the peaks tend to be symmetric around the reconstructed curves and thus their effect is cancelled out.

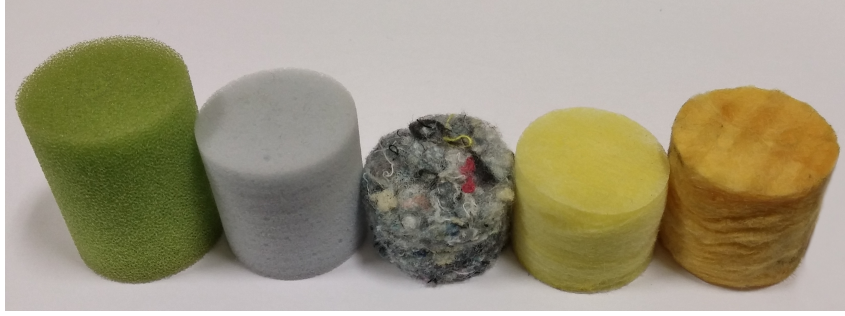


Figure 1: Samples used in this work. From the left: 1. Soft polyurethane foam (SPF), 2. Melamine foam, 3. Felt, 4. Glass wool, 5. Isover Calibel wool.

Table 1: Recovered values for the tested materials using the method proposed in this paper, the method proposed in Refs. (16) and (17) (Olny&Panneton), and direct measurements. O&P values for α_∞ , Λ and σ are calculated with their extrapolation method, except where indicated. For this method also the flow resistivity σ is required as a prior knowledge. The σ for O&P is calculated as the low frequency limit of the real part of the dynamic resistivity.

| Material | | | | | | | |
|----------------------------|---------|--------|-----------------|-----------------------------|------------------------------|--------------------------------|---------------------------------|
| L (mm) | Method | ϕ | α_∞ | Λ (μm) | Λ' (μm) | σ (Nsm^{-4}) | σ' (Nsm^{-4}) |
| SPF 39.8 ± 0.2 | Present | 1.00 | 1.04 | 275 | 491 | 2060 | 1420 |
| | O&P | - | 1.07 | 329 | 448 | 1810 | 1480 |
| | Meas | 0.98 | 1.05 | 289 | $3\Lambda = 867$ | 1800 | - |
| Melamine 31.3 ± 0.1 | Present | 1.00 | 1.00 | 137 | 178 | 7570 | 4550 |
| | O&P | - | 0.79 * | 67 | 166 | 7480 | 4070 |
| | Meas | 0.97 | 1 | 166 | $3\Lambda = 498$ | 8600 | - |
| Felt 16.5 ± 0.5 | Present | 1.00 | 1.00 | 36 | 110 | 25500 | 9570 |
| | O&P | - | 0.77 * | 33 | 118 | 21500 | 8770 |
| | Meas | 1.03 | 1 | 45 | $2\Lambda = 90$ | 28000 | - |
| Wool 22.6 ± 0.3 | Present | 0.93 | 1.00 | 59 | 140 | 32400 | 20200 |
| | O&P | - | 1.16 ** | 53 | 190 | 27000 | 11900 |
| | Meas | 1.01 | 1 | 64 | $2\Lambda = 128$ | 24000 | - |
| Calibel 28.3 ± 0.3 | Present | 0.97 | 1.00 | 36 | 120 | 47800 | 29400 |
| | O&P | - | 1.38 ** | 26 | 91 | 38500 | 16900 |
| | Meas | 0.97 | 1.01 | 37 | $2\Lambda = 74$ | 57000 | - |

* This is obviously an unphysical result. Value of $\alpha_\infty = 1$ was used in the calculation of σ' .

** Calculated with the analytical method.

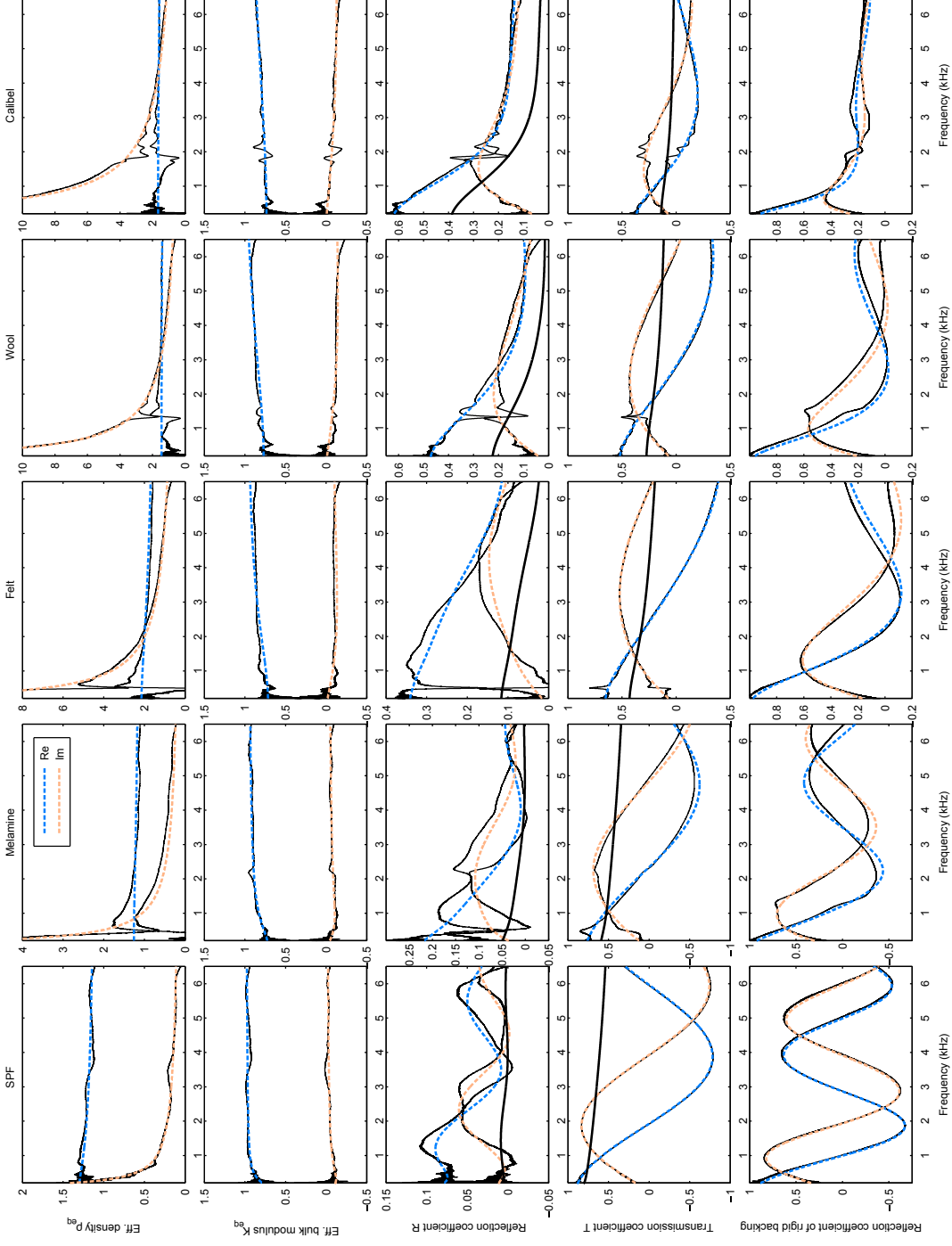


Figure 2: Real and imaginary parts for measured and recovered values for ρ , K , R and T . Thin black lines are the measurements, dotted lines are from the model with the recovered parameters. The bottom row shows a reconstruction and measurement against a rigid backing. The thicker black lines in R and T measurements are $|R|^2$ and $|T|^2$, respectively. Notice that the y -axis changes from figure to figure for better visibility.

REFERENCES

1. M. E. Delany and E. N. Bazley, "Acoustical properties of fibrous materials", *Applied acoustics* **3**, 105-116 (1970).
2. Y. Miki, "Acoustical properties of porous materials -Modifications of Delany-Bazley models-", *J. Acoust. Soc. Jpn. (E)* **11**, 19-28 (1990).
3. J.-F. Allard and N. Atalla, *Propagation of sound in porous media: modelling sound absorbing materials*, 1-358, 2nd edition, John Wiley & Sons Ltd., Chichester, United-Kingdom, (2009).
4. Z. E. A. Fellah, N. Sebaa, M. Fellah, F. G. Mitri, E. Ogam, W. Lauriks and C. Depollier, "Application of the Biot model to ultrasound in bone: direct problem", *IEEE Trans. Ultrason., Ferroelect., Freq. Control* **55**, 1508-1515 (2008).
5. T. G. Zieliński, "Normalized inverse characterization of sound absorbing rigid porous media", *J. Acoust. Soc. Am.* **137**, 3232-3243 (2015).
6. D. L. Johnson, J. Koplik and R. Dashen, "Theory of dynamic permeability and tortuosity in fluid-saturated porous media", *J. Fluid Mech.* **176** 379-402 (1987).
7. Y. Champoux and J. F. Allard, "Dynamic tortuosity and bulk modulus in air-saturated porous media", *J. Appl. Phys.* **70**, 1975-1979 (1991).
8. D. Lafarge, "Propagation du son dans les matériaux poreux à structure rigide saturés par un fluide viscothermique: Définition de paramètres géométriques, analogie électromagnétique, temps de relaxation" (Sound propagation in porous materials saturated with viscothermal fluid: Definition of the geometrical parameters, electromagnetic analogue and relaxation time), Ph. D. Thesis, Université du Maine, Le Mans, France, (1993).
9. S. R. Pride, F. D. Morgan and F. A. Gangi, "Drag forces of porous media acoustics", *Physical Review B* **47**, 4964-4975 (1993).
10. D. Lafarge, P. Lemarinier, J. F. Allard, V. Tarnow, "Dynamic compressibility of air in porous structures at audible frequencies", *J. Acoust. Soc. Am.* **102**, 1995-2006 (1997).
11. D. L. Johnson, J. Koplik and L. M. Schwartz, "New pore size parameter characterizing transport in porous media", *Phys. Rev. Lett.* **57**, 2564-2567 (1986).
12. J. F. Allard and Y. Champoux, "New empirical equations for sound propagation in rigid frame fibrous materials", *J. Acoust. Soc. Am.* **91**, 3346-3353 (1992).
13. J. C. Lagarias, J. A. Reeds, M. H. Wright and P. E. Wright, "Convergence properties of the Nelder-Mead simplex method in low dimensions", *SIAM Journal of Optimization* **9**, 112-147 (1998).
14. L. L. Beranek, "Acoustic impedance of porous materials", *J. Acoust. Soc. Am.* **13**, 248-260 (1942).
15. F. Fohr, D. Parmentier, B. R. J. Castagnede and M. Henry, "An alternative and industrial method using low frequency ultrasound enabling to measure quickly tortuosity and viscous characteristic length", *Acoustics 08 Paris* (2008).
16. R. Panneton and X. Olny, "Acoustical determination of the parameters governing viscous dissipation in porous media", *J. Acoust. Soc. Am.* **119** 2027-2040 (2006).
17. X. Olny and R. Panneton, "Acoustical determination of the parameters governing thermal dissipation in porous media", *J. Acoust. Soc. Am.* **123** 814-824 (2008).
18. Ph. Leclaire, L. Kelders, W. Lauriks, M. Melon, N. Brown and B. Castagnède, "Determination of the viscous and thermal characteristic lengths of plastic foams by ultrasonic measurements in helium and air", *J. Appl. Phys.* **80**, 2009-2012 (1996).