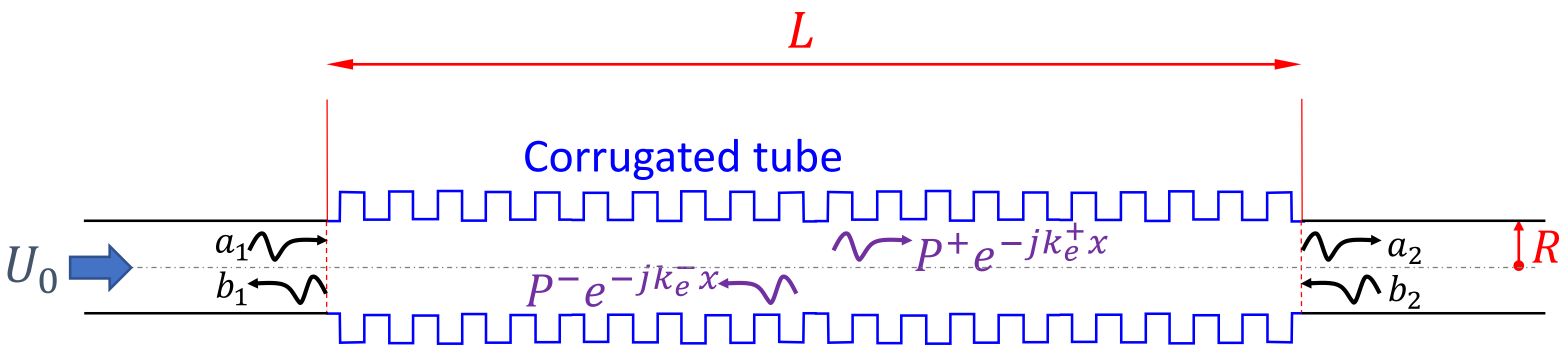
Acoustic Propagation in Corrugated Pipes with flow: Similarities with liners

J. Golliard[a,b]***, T. Humbert***[b]***, Y. Aurégan***[b]

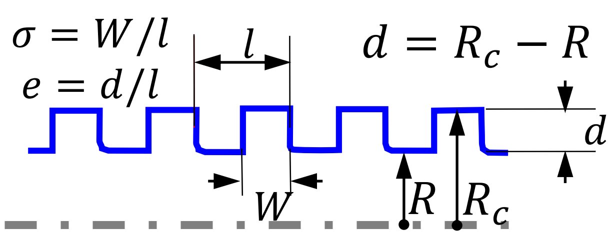
[a] Centre de Transfert de Technologie du Mans
[b] Laboratoire d'Acoustique de l'Université du Mans

Introduction

Geometry of corrugated pipe

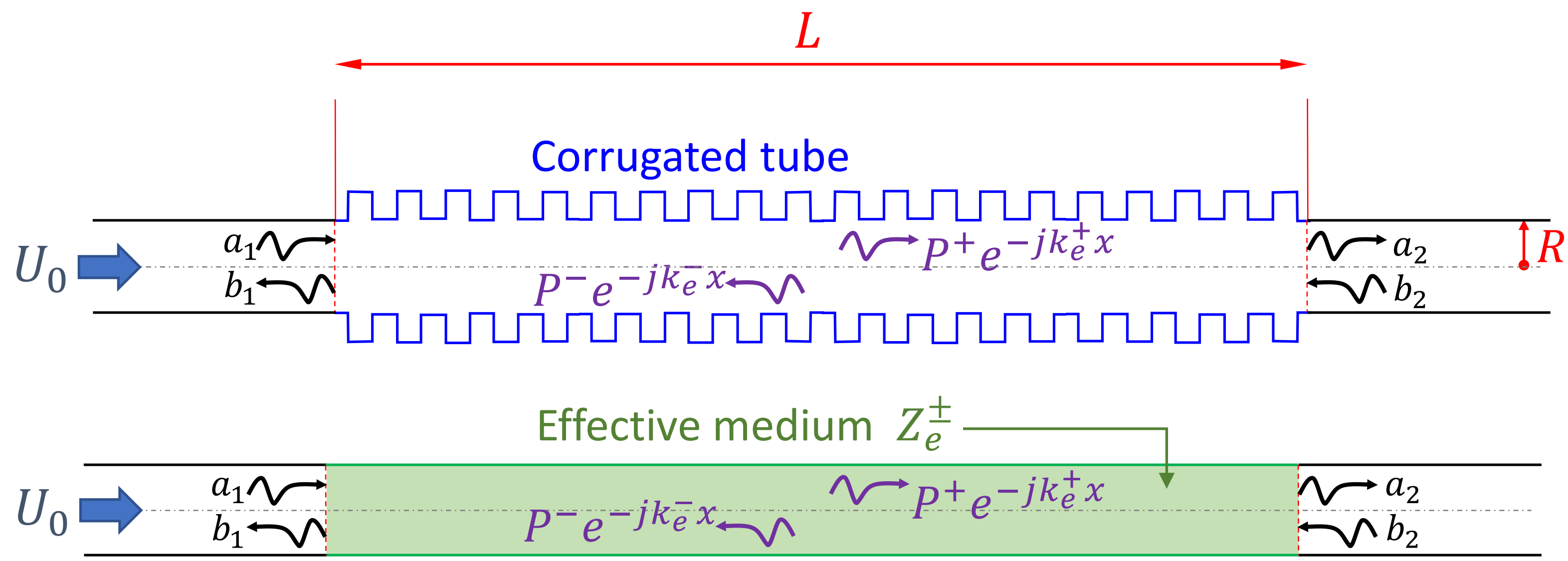


- Axisymmetric corrugated pipe of length $L = 2$ m and diameter 30 mm
- Measurements of scattering matrix with through–flow at different Mach numbers



$R = 15$ mm, $W = 4$ mm
$d = 4$ mm, $l = 12$ mm

Corrugated tube as an effective wall medium



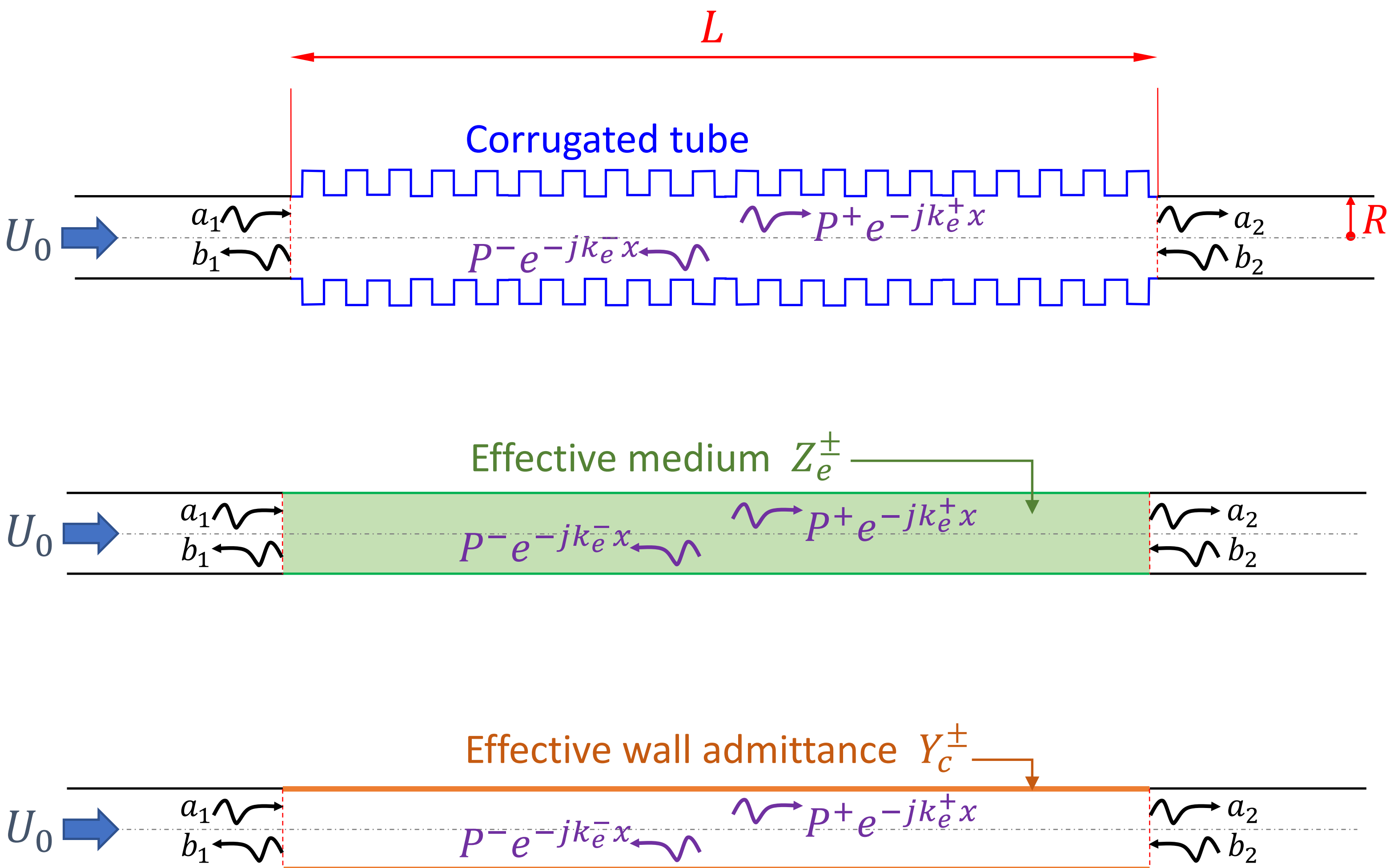
One-dimensional acoustic propagation in the corrugated tube considered to be described by an effective medium:

- $Z_e^\pm$: Effective characteristic impedance
- $\boldsymbol{k_e^\pm}$**: Effective wavenumber**

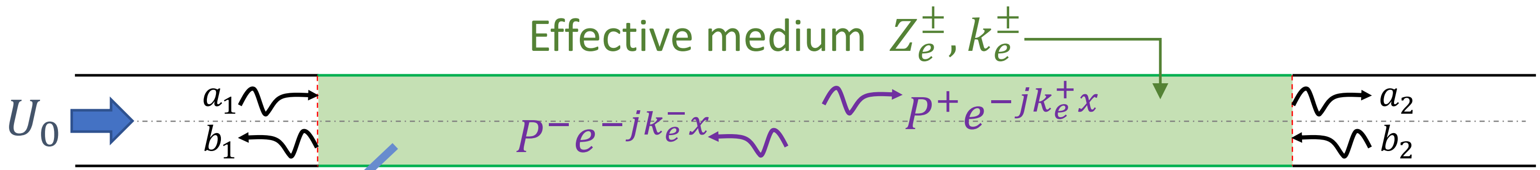
Properties allowed to be direction dependent

$Z_e^\pm$ and $k_e^\pm$ extracted from scattering matrix of corrugated pipe (measured between smooth pipes)

Corrugated tube as an effective wall admittance

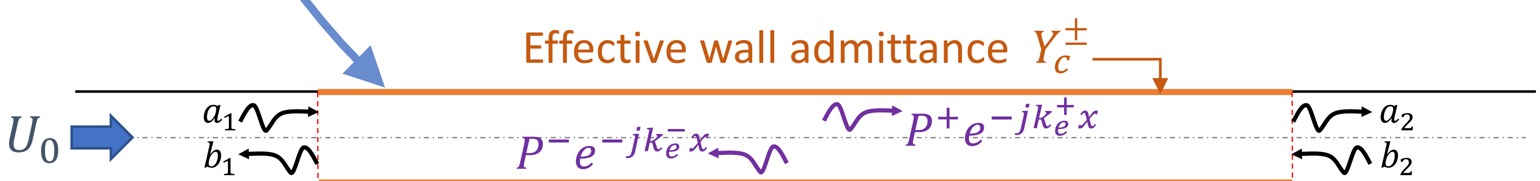


Corrugated tube as an effective wall admittance

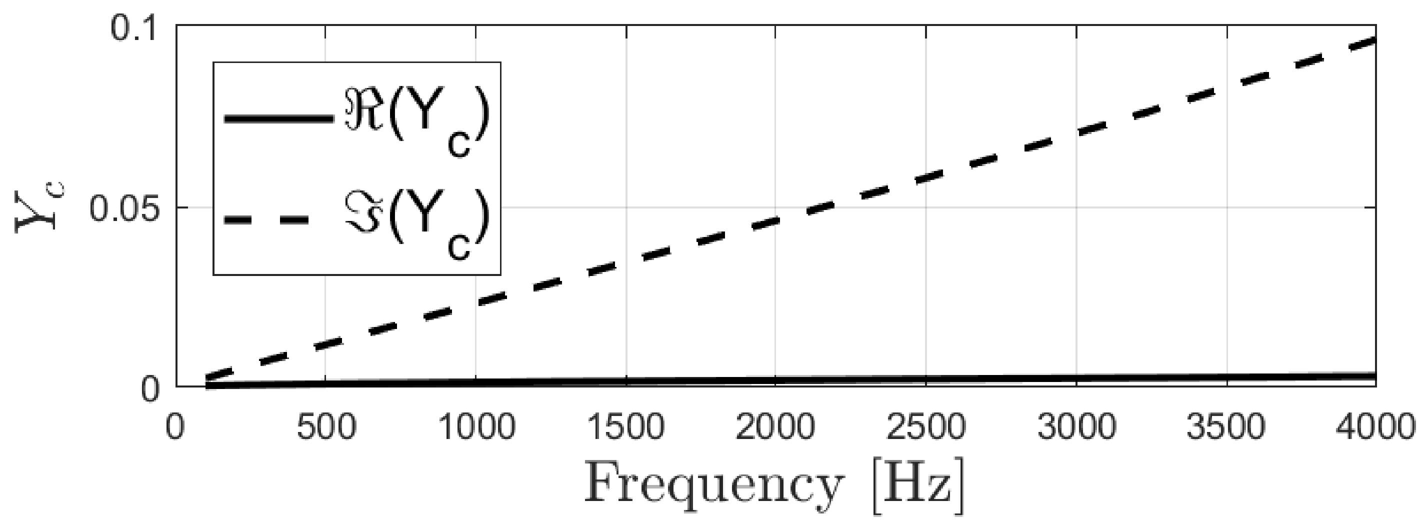


Without flow ($k_e = k_e^+ = k_e^-$):

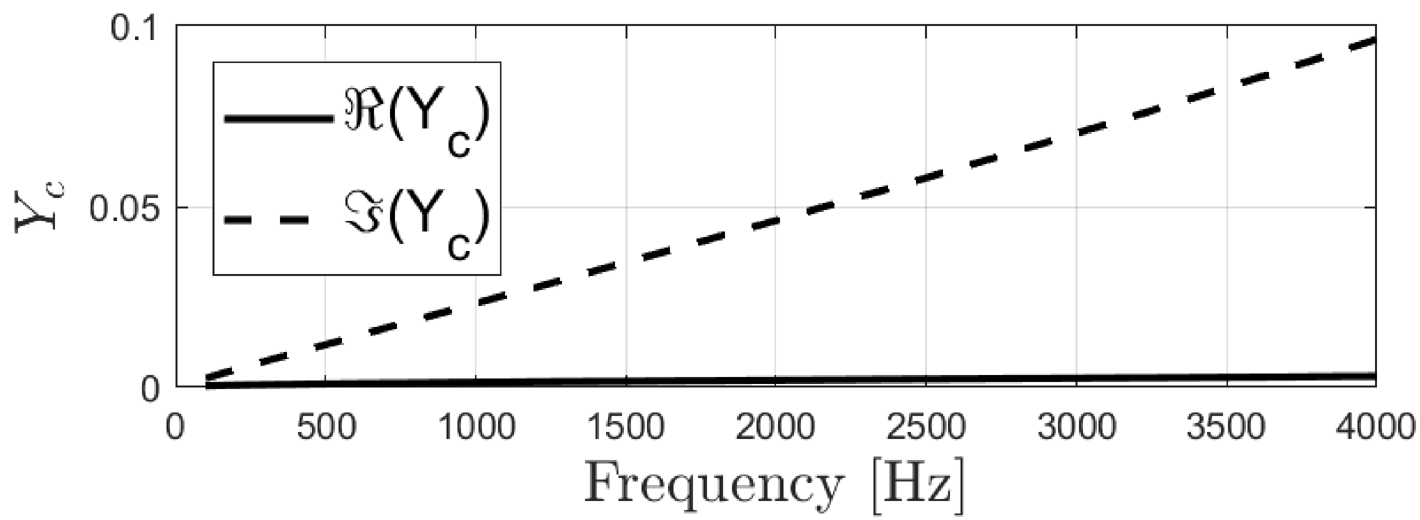
$$Y_c = -jR\frac{\alpha_{\pm} J_1(\alpha r)}{J_0(\alpha r)}, \text{ with } \alpha = \sqrt{k_0^2 - k_e^2}$$



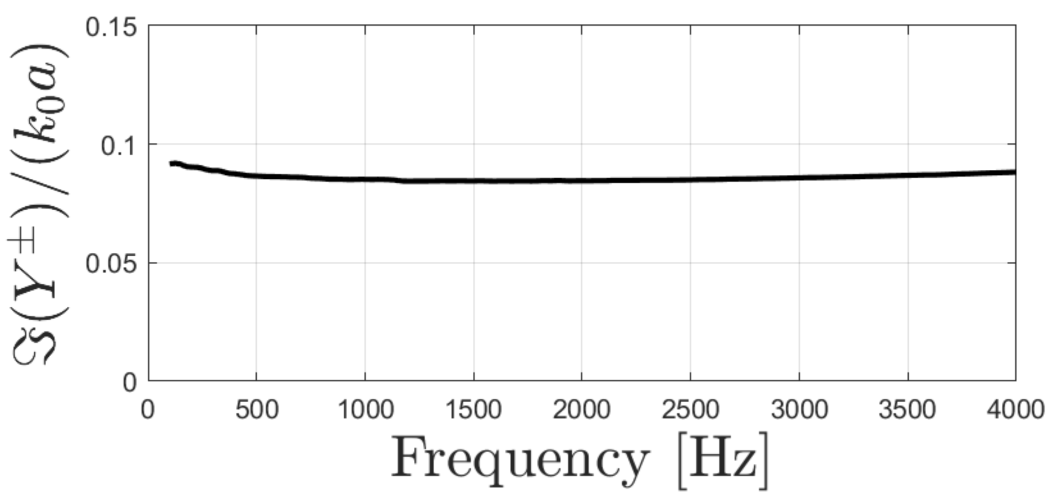
Wall admittance measured without flow



Wall admittance measured without flow



Small but non-negligible compliance of the wall is observed:

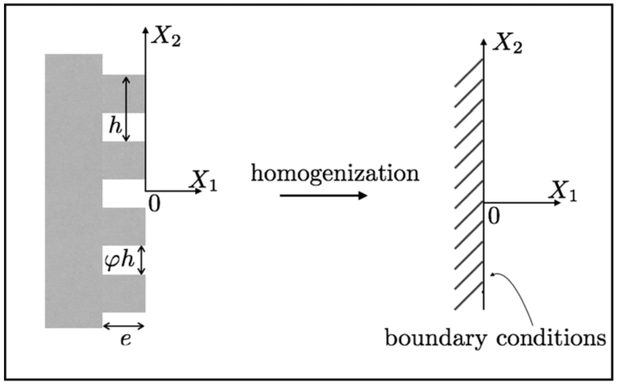


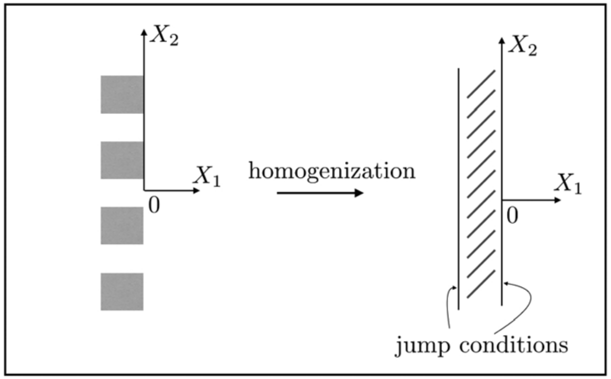
Homogenization models for thin rigid structured surfaces and films

Jean-Jacques Marigo
Laboratoire de Mécanique du Solide, CNRS, Ecole Polytechnique, Route de Saclay, 91128 Palaiseau, France
Agnès Maurel[a)]
Institu Langevin, CNRS, ESPCI ParisTech, 1, rue Jussieu, 7500 Paris, France

(Received 9 February 2016; revised 8 June 2016; accepted 10 June 2016; published online 14 July 2016)

A homogenization method for thin microstructured surfaces and films is presented. In both cases, sound hard materials are considered, associated with Neumann boundary conditions and the wave equation in the time domain is examined. For a structured surface, a boundary condition is obtained on an equivalent flat wall, which links the acoustic velocity to its normal and tangential derivatives (of the Myers type). For a structured film, jump conditions are obtained for the acoustic pressure and the normal velocity across an equivalent interface (of the Ventcels type). This interface homogenization is based on a matched asymptotic expansion technique, and differs slightly from the classical homogenization, which is known to fail for small structuration thicknesses. In order to get insight into what causes this failure, a two-step homogenization is proposed, mixing classical homogenization and matched asymptotic expansion. Results of the two homogenizations are analyzed in light of the associated elementary problems, which correspond to problems of fluid mechanics, namely, potential flows around rigid obstacles. © *2016 Acoustical Society of America*.
[http://dx.doi.org/10.1121/1.4954756]



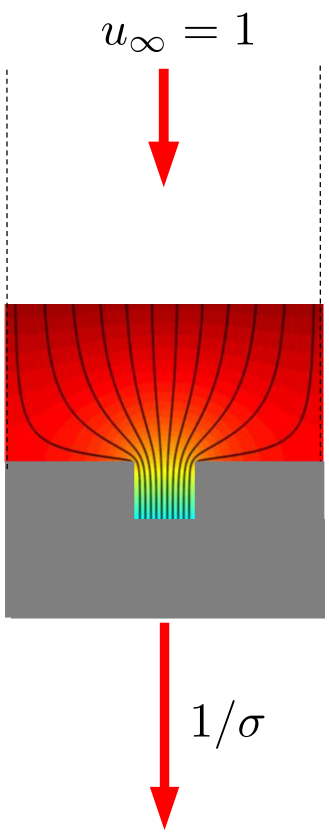


Order 1 solution

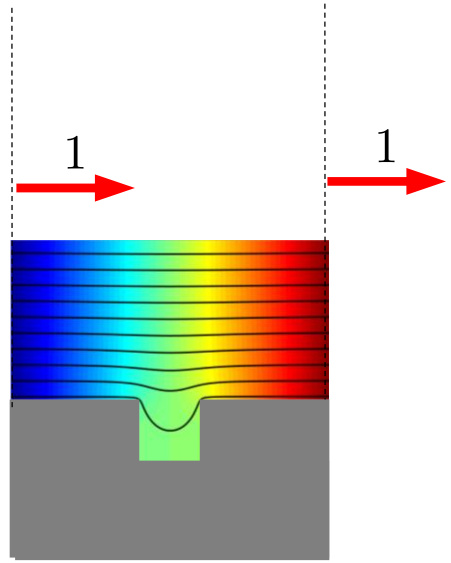
$$P^1 = \frac{\partial p^0}{\partial x_1}(0, x_2)P^A(\vec{y}) + \frac{\partial p^0}{\partial x_2}(0, x_2)P^B(\vec{y}) + \tilde{P}(\vec{x})$$

2 elementary problems

Problem A

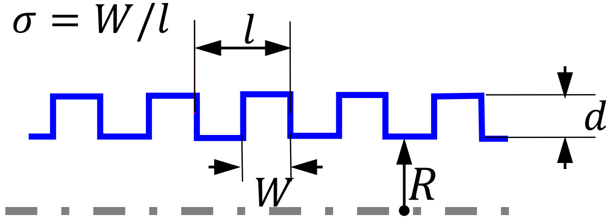


Problem B



By definition, the problem B is not considered with the classical homogenization process

Equivalent admittance of 2D corrugations to incident field



Equivalent boundary condition computed by homegenization of a periodic latice of 2D cavities:

$$v(x,R) = d\sigma\partial_r v\Big|_{r=R} + lC\partial_x u\Big|_{r=R}$$

with C computed from incompressible problem in longitudinal direction.

In 2D with rigid wall at R in front of corrugation, $p = P\cos(\alpha r)$,

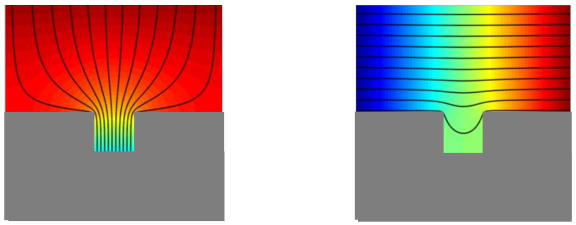
leading to $Y_c = \frac{v(R)}{p(R)} = jk_0R\left(\frac{d}{R}\sigma\left(1-\left(\frac{k}{k_0}\right)^2\right) + \frac{lC}{R}\left(\frac{k}{k_0}\right)^2\right)$

$\left(\frac{k}{k_0}\right)^2 \sim 1$ => compressibility term will be much smaller than inertance term.

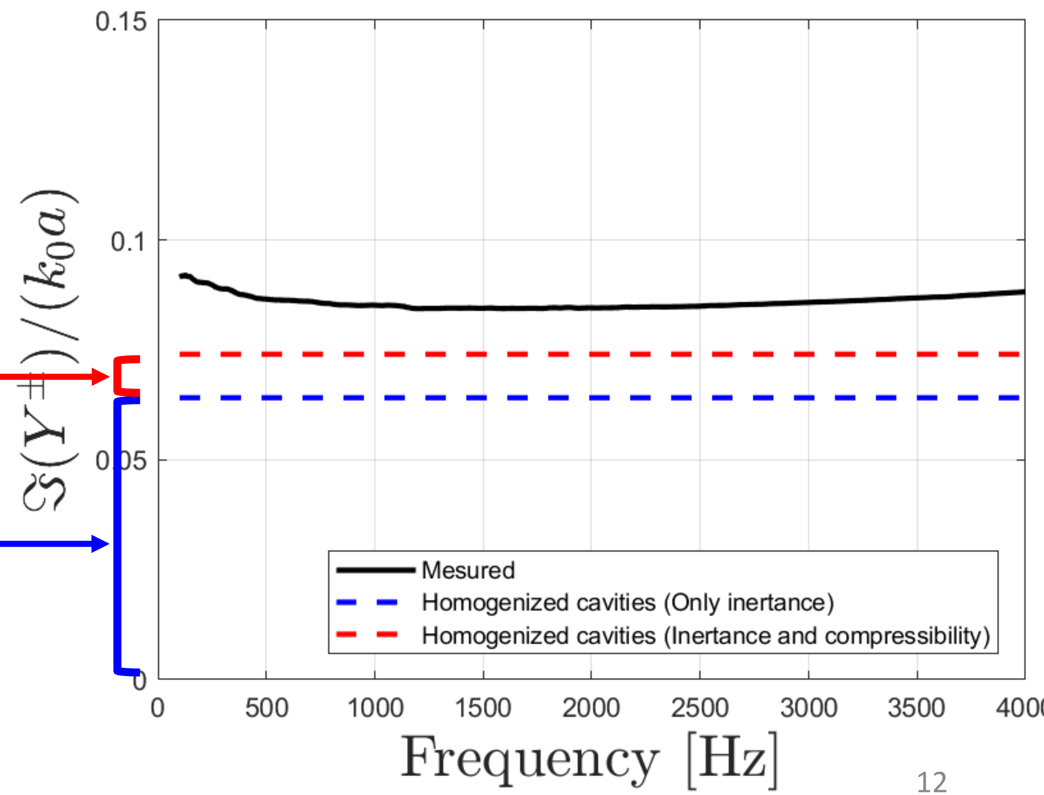
Actually: Compressibility: $Y_{c_{Comp.}} \sim jk_0R\,\frac{2\sigma Cdl}{R^2+dR\sigma} = 0.01$

Inertance: $Y_{c_{Inert.}} \sim jk_0R\;lC\,\frac{R+d\sigma-2lC}{R^2+dR\sigma} = 0.064$

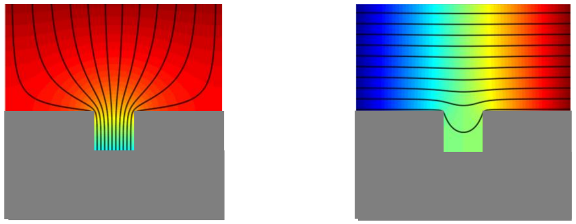
Wall admittance measured without flow



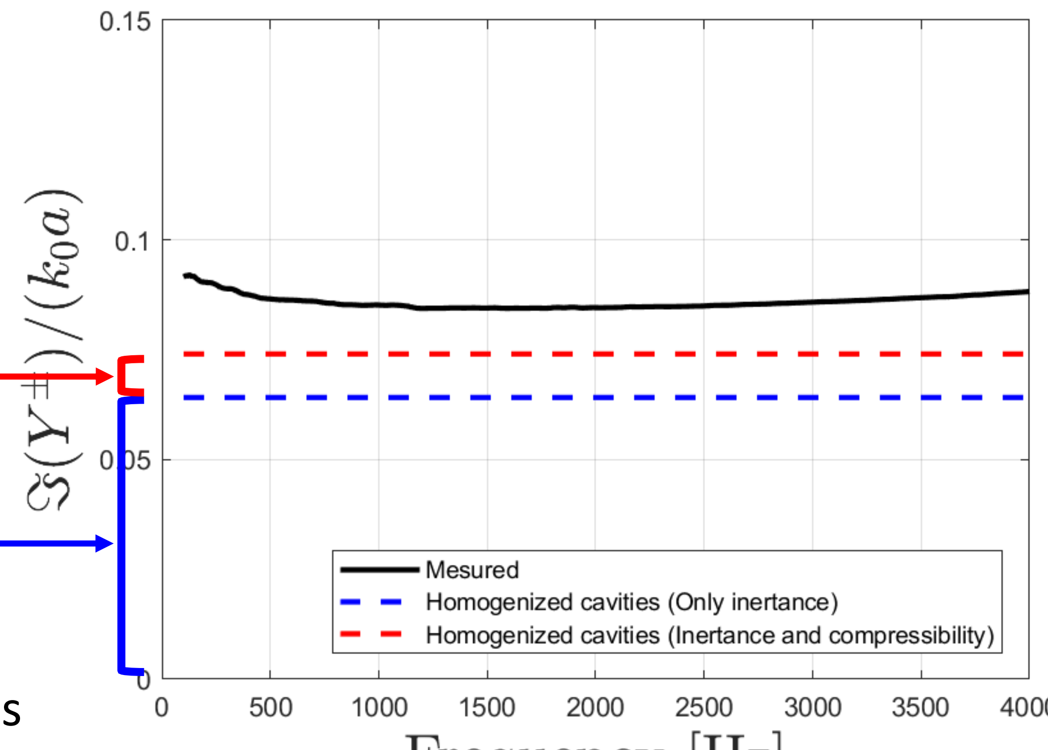
$$v(x,R) = d\sigma\partial_r v\Big|_{r=R} + lC\partial_x u\Big|_{r=R}$$



Wall admittance measured without flow

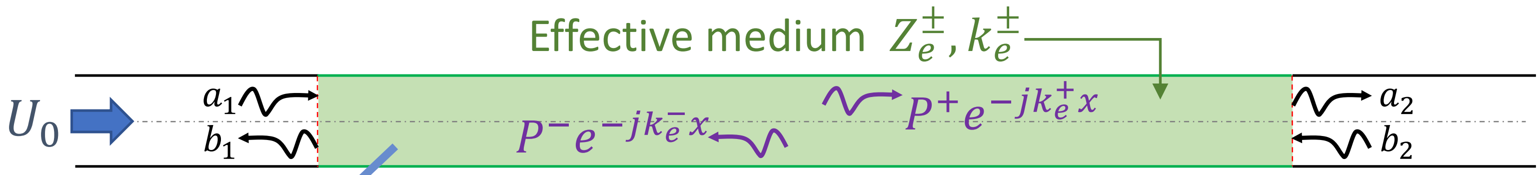


$$v(x,R) = d\sigma\partial_r v\Big|_{r=R} + lC\partial_x u\Big|_{r=R}$$



The classical homogenization (and the way we understand impedance or admittance) forgets the most important contribution

Corrugated tube as an effective wall admittance

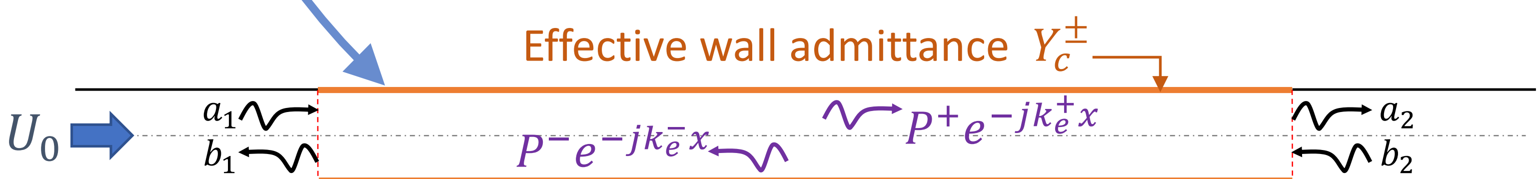


Without flow:

$$Y_c = -jR\frac{\alpha_{\pm}J_1(\alpha r)}{J_0(\alpha r)}, \text{ with } \alpha = \sqrt{k_0^2 - k_e^2}$$

With flow: Ingard–Myers Boundary Condition:

$$Y_c^{\pm} = -jR\frac{k_0^2}{\Omega^{\pm 2}}\frac{\alpha_{\pm}J_1(\alpha_{\pm} r)}{J_0(\alpha_{\pm} r)}, \text{ with } \alpha_{\pm} = \sqrt{\Omega^{\pm 2} - k_e^{\pm 2}}, \Omega^{\pm} = k_0 - M_0 k_e^{\pm}$$



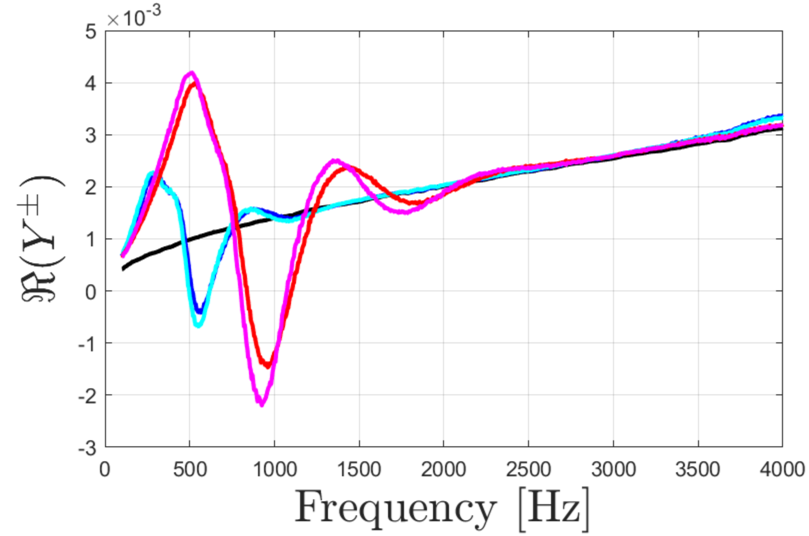
Wall admittance measured with flow

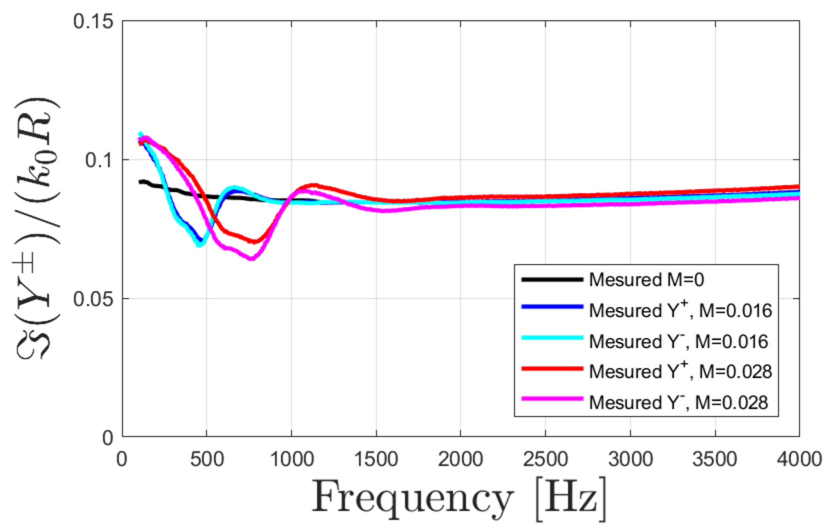
Oscillations:

- Large oscillations of both $\Re(Y_c^{\pm})$ and $\Im(Y_c^{\pm})$
- Frequency of oscillations dependant on flow velocity: constant Sr
- Frequency range where $\Re(Y_c^{\pm}) < 0$: wall acts as sound amplifier
- If conditions are (un)favorable, whistling will occur at these frequencies

But also:

- $Y_c^{\pm}$ is not unique: $Y_c^{+} \neq Y_c^{-}$
- Behavior similar to liners

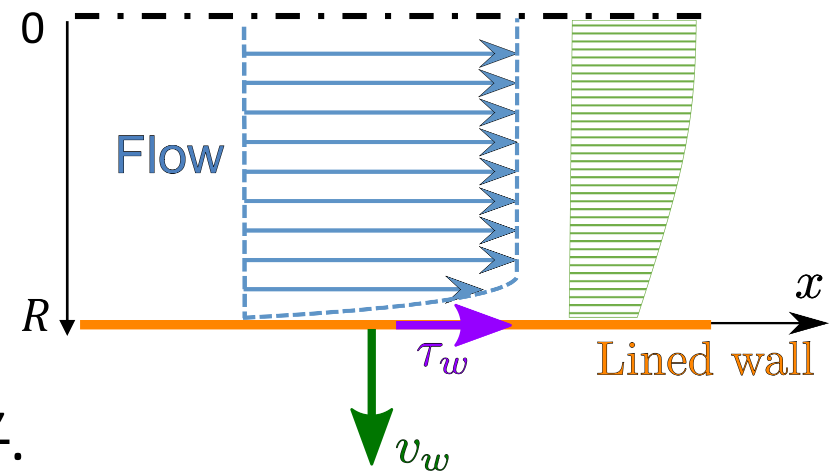




Stress–Impedance model

$$Y_c^{\pm} = Y_\tau + \frac{k_e^{\pm}}{\omega} f_\tau,$$

where Y_τ is the admittance of the lined wall and f_τ is a friction coefficient depending on the pressure at the wall $f_\tau = \frac{\tau_w}{p_w}$.



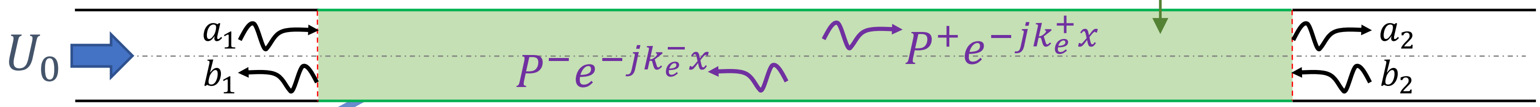
τ_w describes an unsteady transfer of momentum from the flow into the wall due to cavities and to turbulent effects.

From two different values $k_e^{\pm}$, we obtain two different values $Y_c^{\pm}$, and then we can computed Y_τ and f_τ :

$$f_\tau = \frac{k_0}{k_e^+ - k_e^-}(Y_c^+ - Y_c^-),\ Y_\tau = Y^+ - \frac{k_e^+}{k_0} f_\tau$$

Effective wall admittance with stress-impedance model

Effective medium $Z_e^{\pm}, k_e^{\pm}$



Ingard–Myers Boundary Condition:

$$Y_c^{\pm} = -j \frac{k_0}{\Omega^{\pm 2}} \frac{\alpha_{\pm} J_1(\alpha_{\pm} r)}{J_0(\alpha_{\pm} r)}, \text{ with } \alpha_{\pm} = \sqrt{\Omega^{\pm 2} - k_e^{\pm 2}}, \Omega^{\pm} = k_0 - M_0 k_e^{\pm}$$

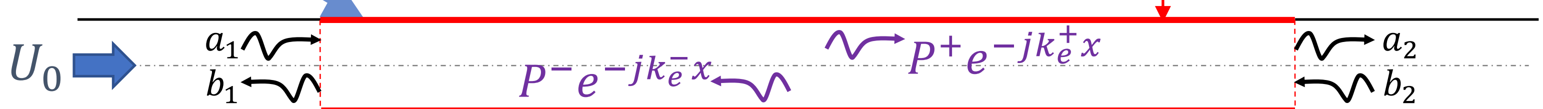
Effective wall admittance $Y_c^{\pm}$

Stress impedance model:

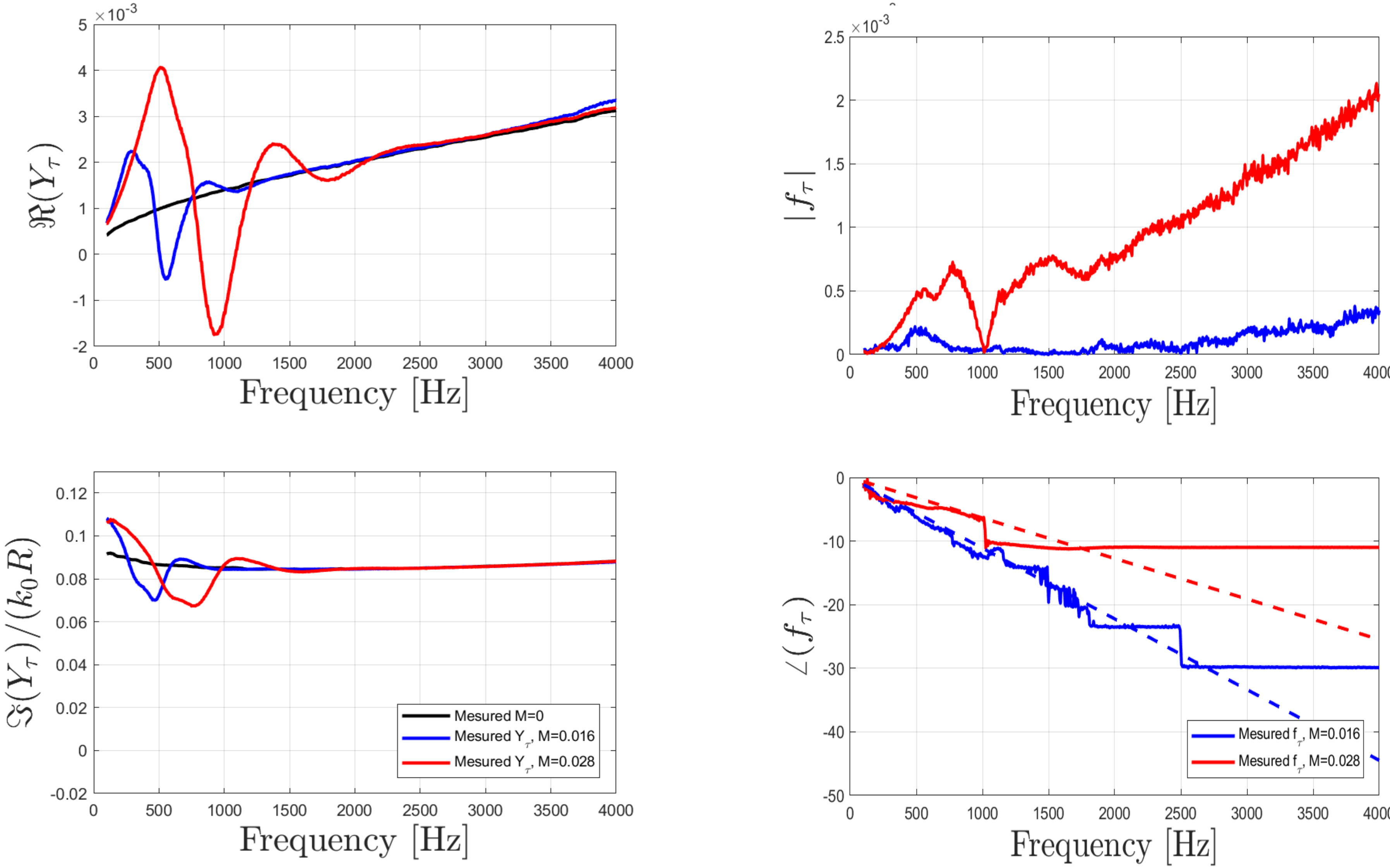
$$Y_c^{\pm} = Y_\tau + \frac{k_e^{\pm}}{\omega} f_\tau, \text{ leads to } \quad f_\tau = \frac{k_0}{k_e^+ - k_e^-}(Y_c^+ - Y_c^-)$$

$$Y_\tau = Y^+ - \frac{k_e^+}{k_0} f_\tau$$

Effective wall admittance Y_τ and friction coefficient f_τ



Wall admittance measured with flow: Stress impedance model



Conclusions

- Using methods describing propagation in treated ducts for corrugated pipes:

  Without flow:

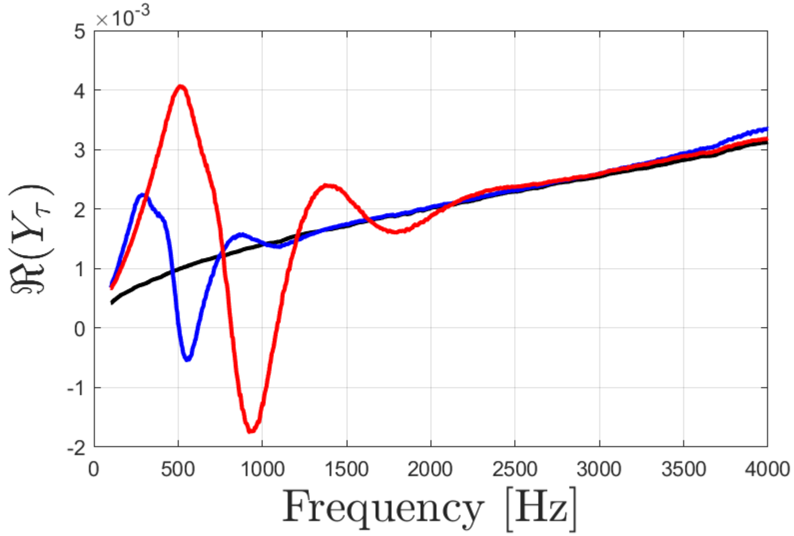
  - Gives insight on how cavities change admittance of the wall
  - Homogenization should not only consider compressibility as could be assumed intuitively

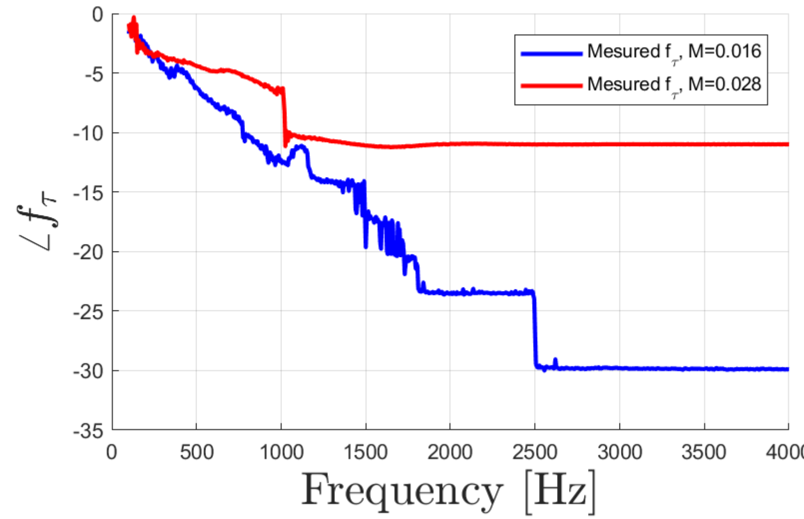
  With flow:

  - The "source of sound" corresponds to negative real admittance of equivalent pipe wall
  - Unique wall admittance does not exist: 2 quantities are required, for example with stress-impedance model

    => friction coefficient seems to indicate that friction changes with delay of convected vorticity

Thank you for your attention



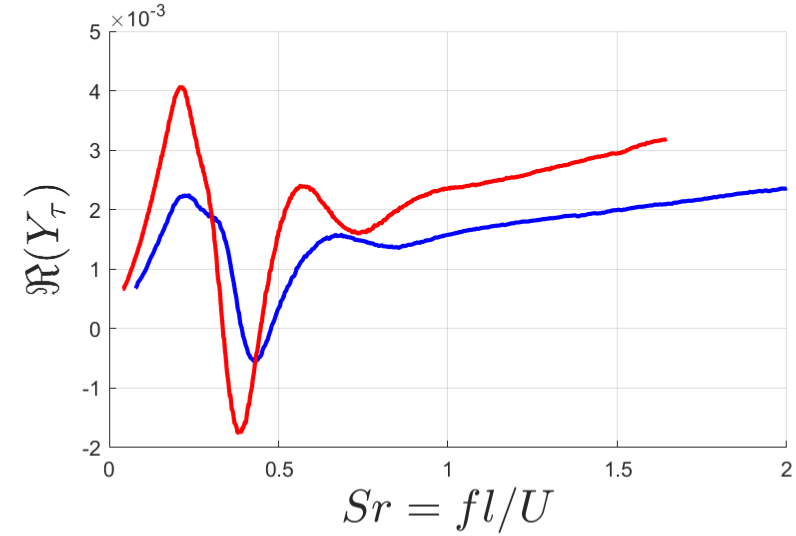


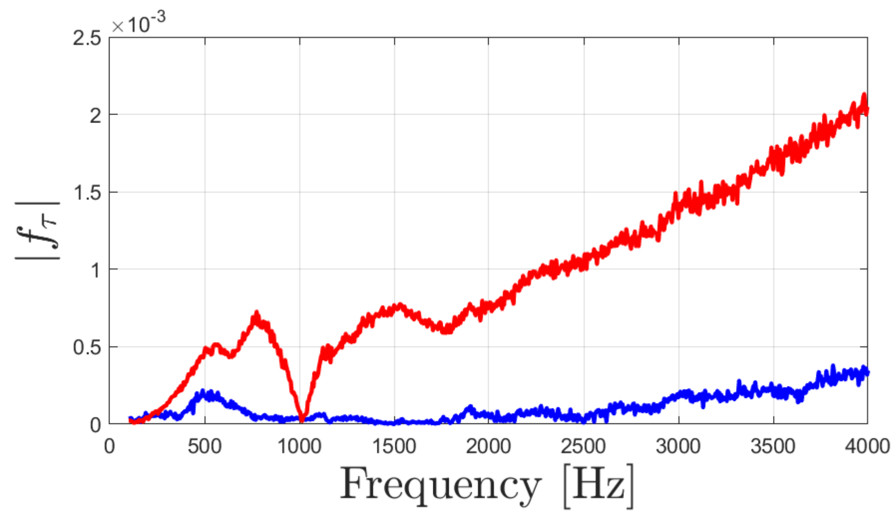
J. Golliard[a,b], T. Humbert[b], Y. Aurégan[b]

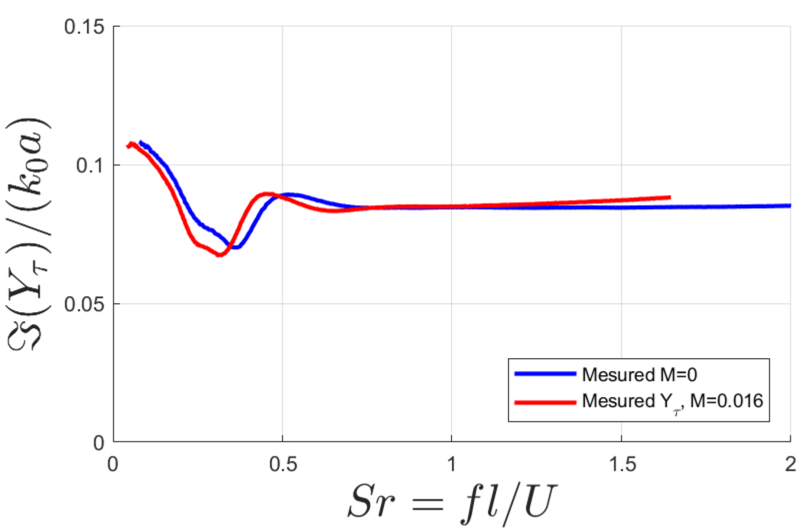
[a] Centre de Transfert de Technologie du Mans

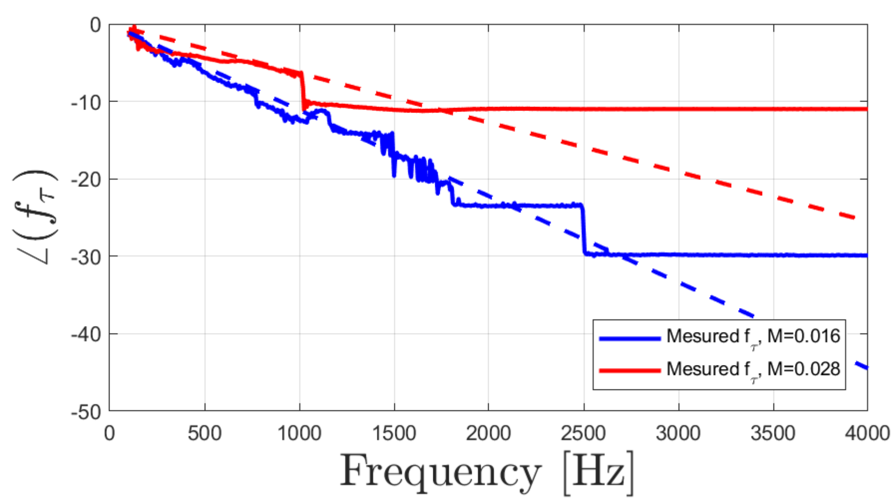
[b] Laboratoire d'Acoustique de l'Université du Mans

Wall admittance measured with flow: Stress impedance model

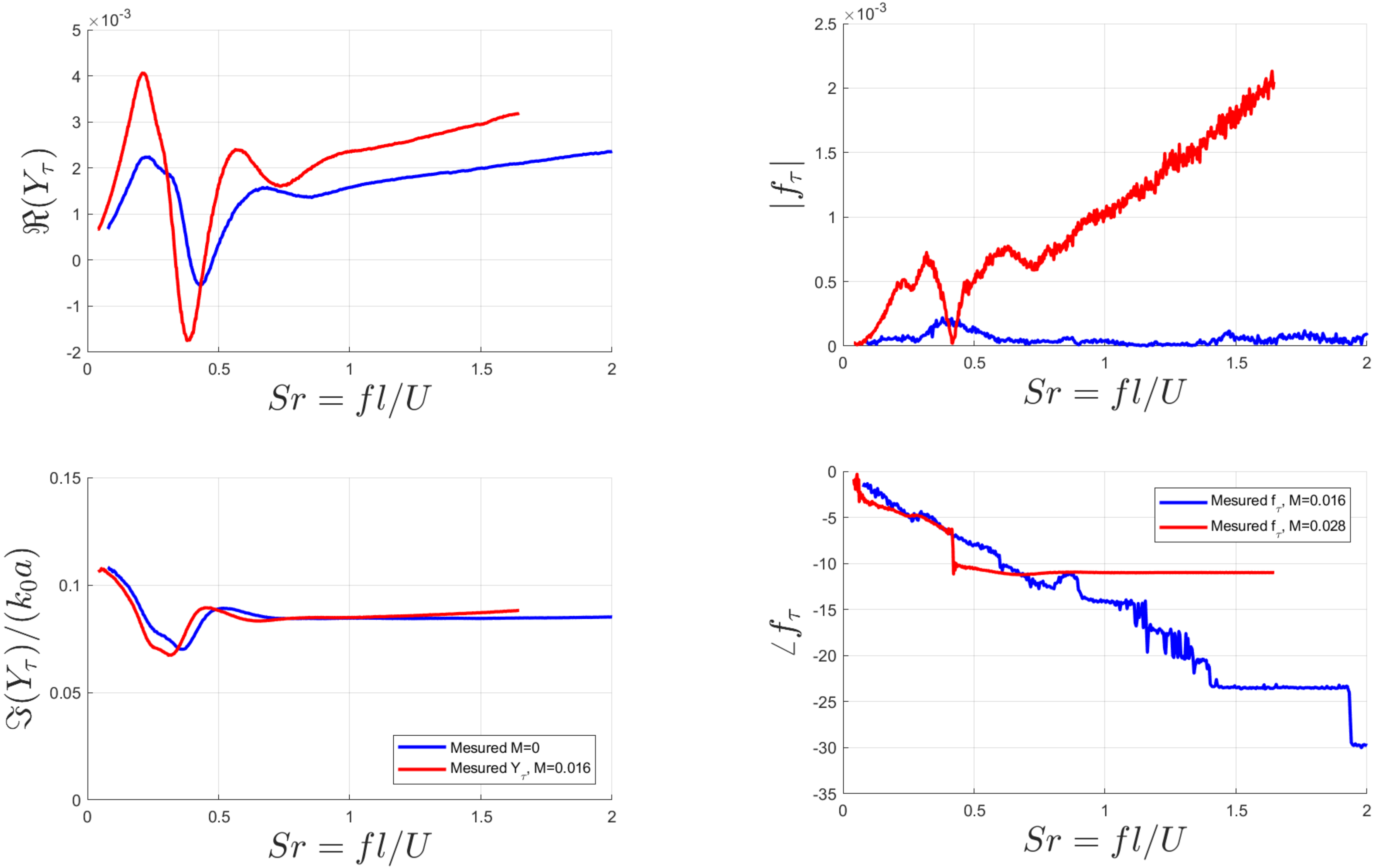






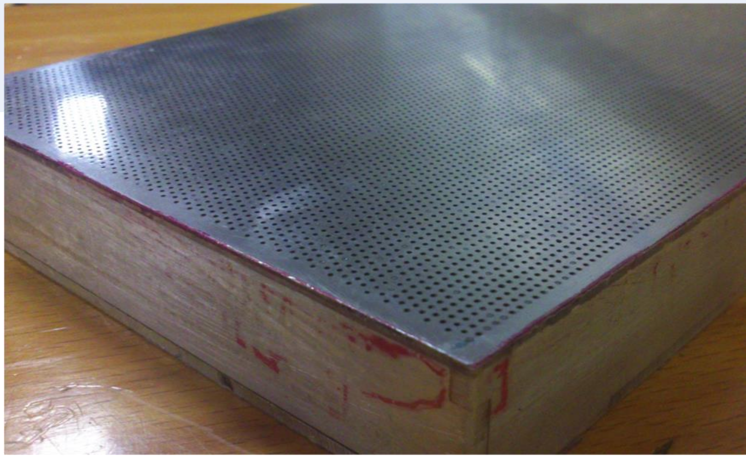


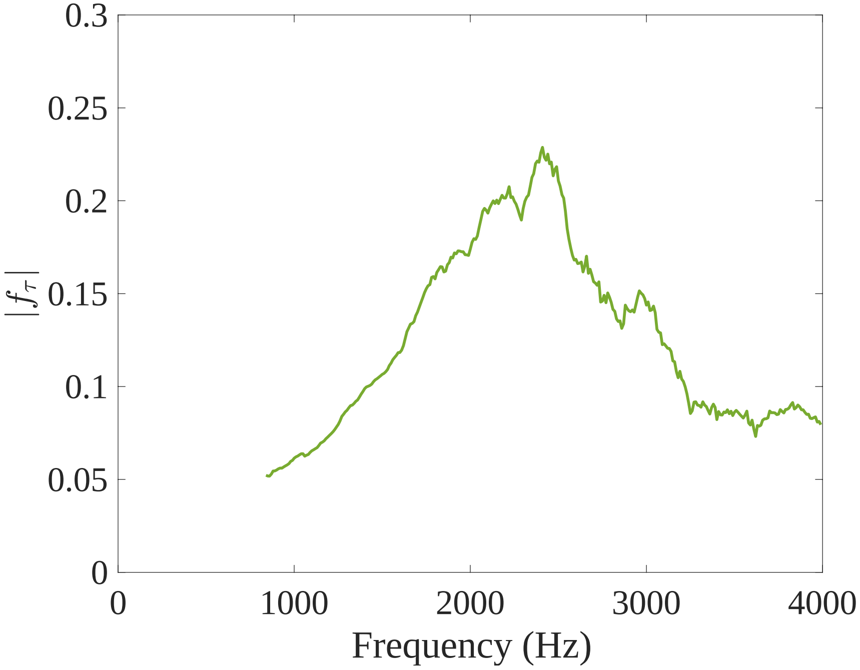
Wall admittance measured with flow: Stress impedance model

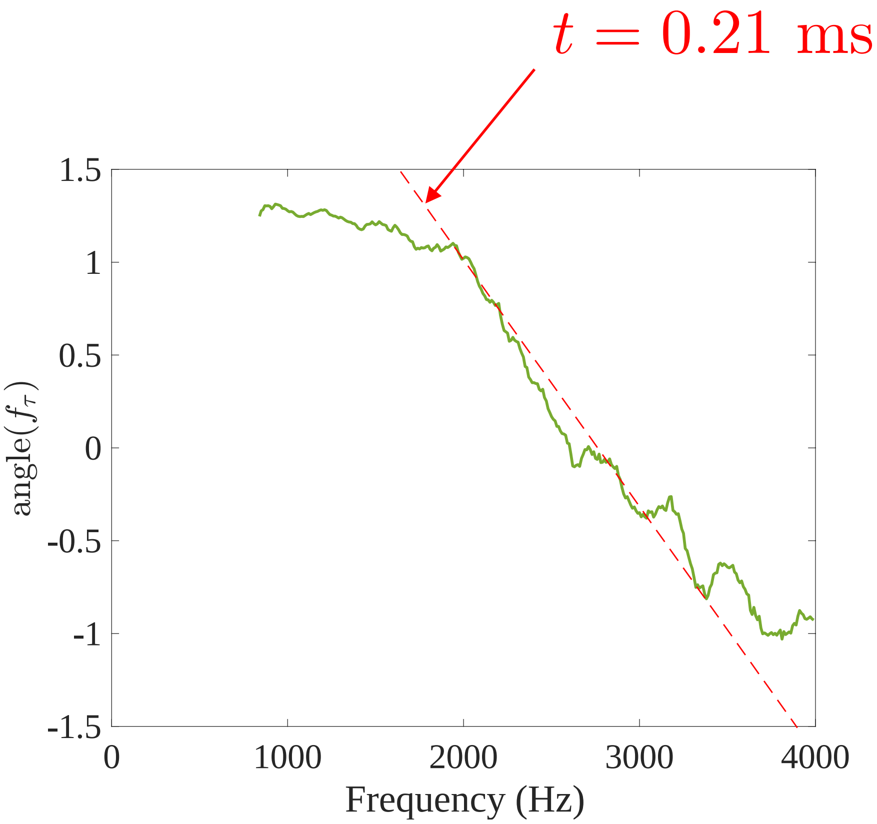


Stress–Impedance results

Micro-perforated Liner

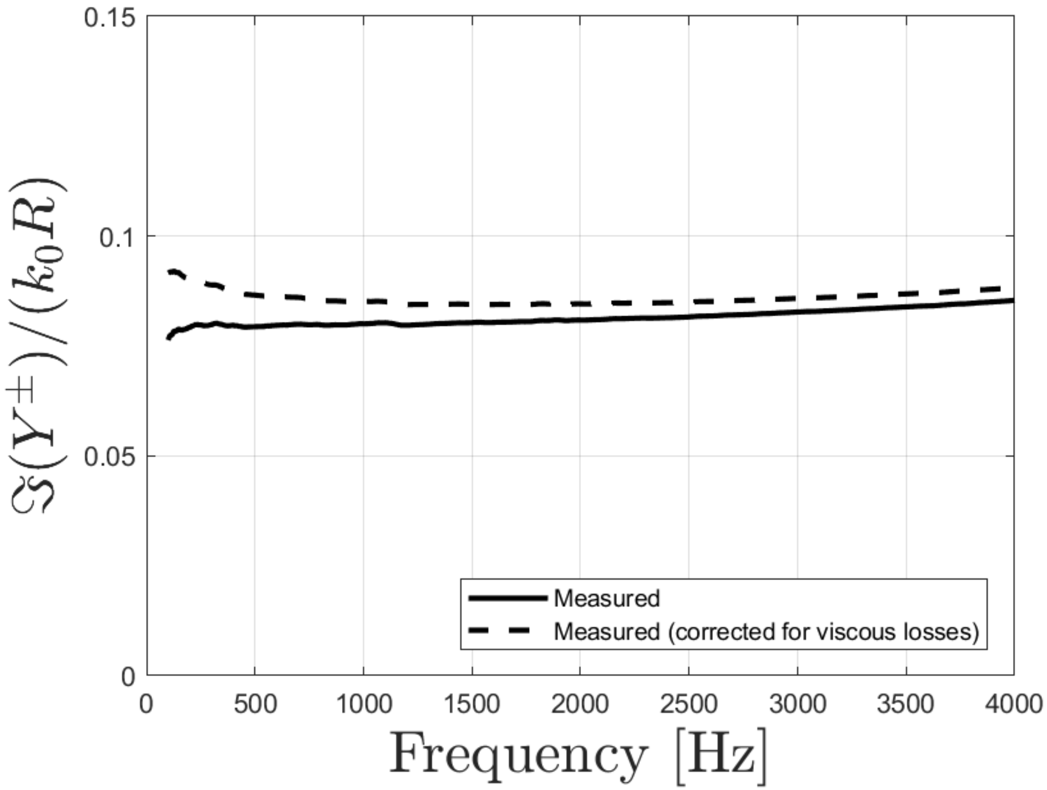




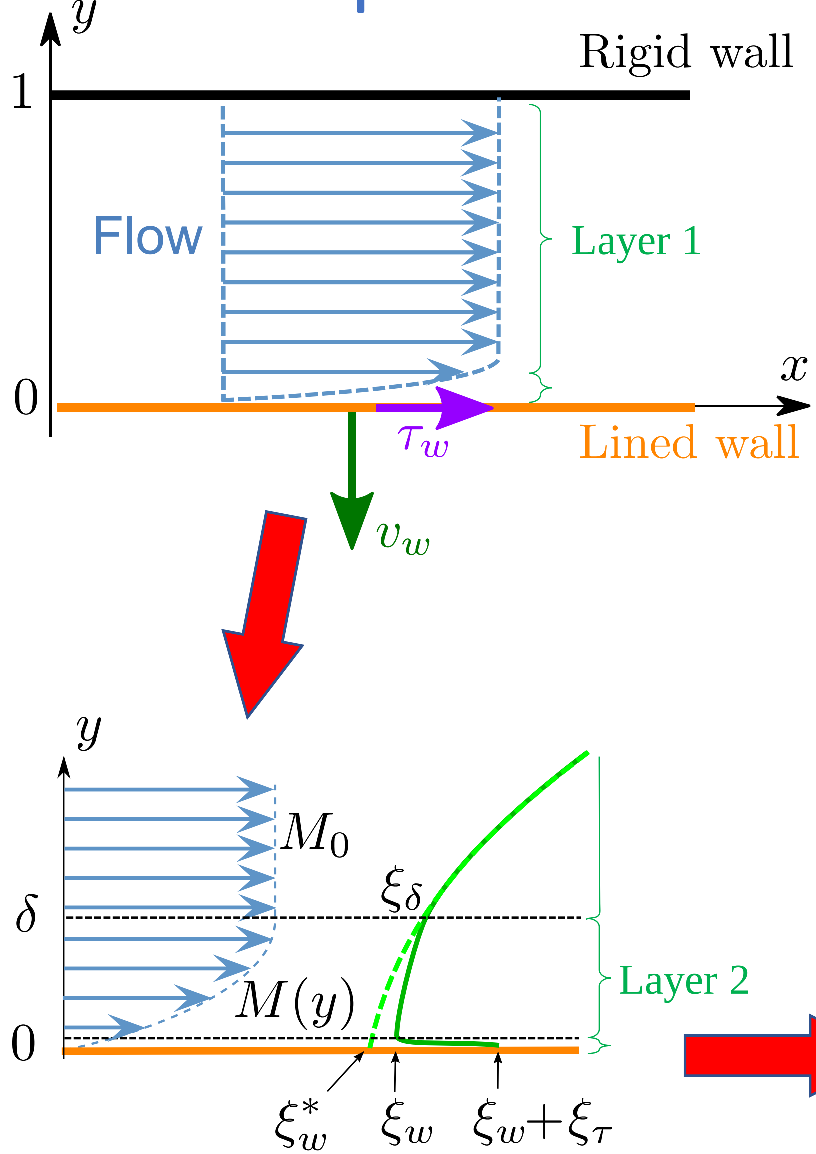


Wall admittance measured without flow

- The visco-thermal losses ($\Im(k_e)$) have a small effect on the speed of sound ($\Re(k_e)$)
- This is also seen on $\Im(Y)$
- If v-t losses are removed from k_e, measurement "corresponds to the inviscid model"



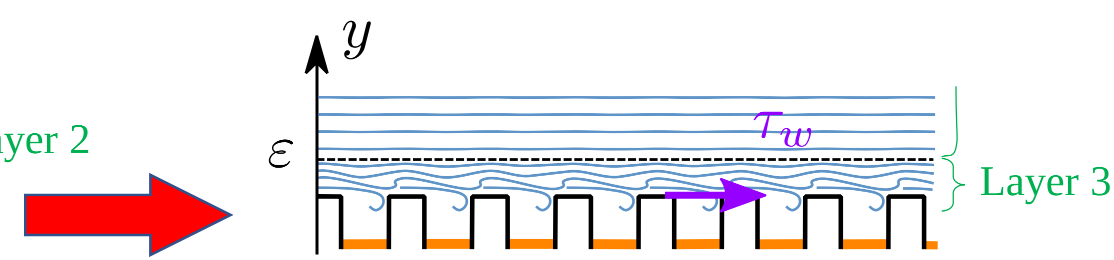
Stress–Impedance model



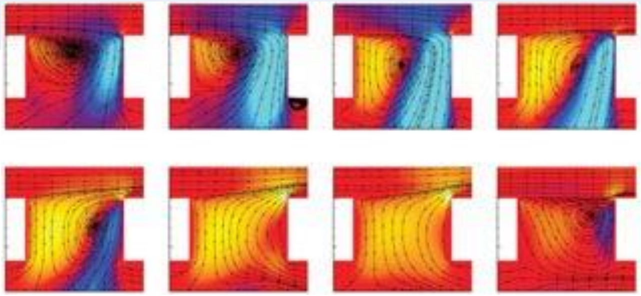
To the first order in δ/λ, the variation of the pressure and of the vertical displacement in the boundary layer can be neglected

$$\xi_w^* = \xi_w$$

$$p_w = p_w^*$$



Stress–Impedance model



From *Zhang & Bodony*, (JFM 2016)

τ_w is intended to describe an unsteady transfer of momentum from the flow into the wall due to wall holes and to turbulent effects.

Integrating the momentum eq. in the Layer 3 leads to:

$$\omega^2 \xi_\tau = \mathrm{j} k \tau_w$$

Stress–Impedance model

The equivalent admittance $Y_w^* = -\frac{v_w^*}{p_w^*}$,

seen by a wave propagating in a uniform flow,

can be computed from the admittance of the wall Y_w

and the friction coefficient defined by $f_w = \frac{\tau_w}{p_w}$

$$Y_w^* = \frac{\Omega_0}{\omega}\left(Y_w + \frac{k}{\omega} f_w\right)$$