Acoustic propagation in pipes with corrugated treatment

Joachim Golliard*
CTTM Le Mans, 20 rue Thalès de Milet, 72000 Le Mans, France

Yves Aurégan and Thomas Humbert
LAUM - Laboratoire d'Acoustique de l'Université du Mans, Le Mans Université, France

I. Introduction

This paper considers linear acoustic propagation in a corrugated pipe with flow. Additionally to the interesting features of corrugated pipes with flow (amplification of sound waves, possibility to sing under certain conditions, etc.), the main topic of interest here is the analogy which can be drawn with the behavior of treatments such as acoustic liners in presence of flow.

Corrugated pipes are found in many industrial applications where a global flexibility is required (to allow relative movements of the parts or devices connected to either end of the pipe) while local strength is also needed (for example to avoid a collapsing pipe when external pressure is applied). Typical examples are vacuum cleaner hoses or flexible pipes for the transport of natural gas on off-shore installations. An interesting feature of such pipes is that they can "sing" under certain conditions, particularly for high through flow [1, 2]. The onset of this singing is a subject of interest in recent years, which requires accurate measurements of the propagation in the linear regime before the (non-linear) singing regime is reached. However, these measurements have shown some interesting analogies with the acoustics of liners used to reduce the acoustic emissions of aircraft engines. Particularly, it appears that the acoustic propagation in the corrugated is different for a wave traveling in the same direction as the mean flow compared to a wave travelling direction opposite to the mean flow.

In this paper, experiments are reported where the diffraction matrix of a corrugated pipe is measured without mean flow and in presence of a mean flow through the pipe. Furthermore, as is commonly done for acoustic treatments, an equivalent impedance of the corrugations is extracted using the Ingard-Myers condition. A typical feature of corrugated pipes with flow is that the acoustic wave interacts with the shear layers, which causes an additional attenuation (or amplification) of theses waves. This translates as a negative conductance. Last, the Stress-Impedance model is used to explain the small difference observed between the admittance measured for acoustic propagation in the direction of the main flow or in the opposite direction [3].

II. Propagation in a corrugated pipe as an equivalent treated pipe

In this section, the propagation in the corrugated pipe is modeled as the propagation in a pipe with a wall impedance equivalent to the corrugations. This will allow to compute the equivalent wall admittance from the knowledge of the measured acoustic wavenumber in the corrugated pipe.

The different steps of modeling are illustrated in Figure 1. The acoustic wavenumber in the corrugated pipe, which is considered an input in this section, is already a model of the pipe, where it is considered that the acoustic propagation in the corrugation of the pipe can be described as the propagation in an equivalent material. This is a common way to model propagation in a treated pipe, but requires an additional step to extract the equivalent wavenumber from experimental data. A continuous model of the corrugated pipe and of the smooth measurement pipes is used to compute the equivalent wavenumber from the coefficients of the diffraction matrix [4]. This step is not detailed here. The second step is to compute the equivalent wall admittance from the wavenumber, which is the subject of Section II-A. An additional step is also suggested in Section II-B, which is allowed by the introduction of the stress-impedance model [5]. It consists in adding a term to the equivalent wall admittance. This term accounts for the additional friction due to the exchange of momentum from the flow to the wall due to the turbulence of to the organized vorticity.

A. Corrugations as a wall admittance

An assumption made here is that any effect of the velocity gradient is concentrated in a thin boundary layer of thickness δ. Outside this layer, a uniform mean flow at Mach number $M_0 = U_0/c_0$ is present and the propagation

*Associated researcher at LAUM, Le Mans Université, France

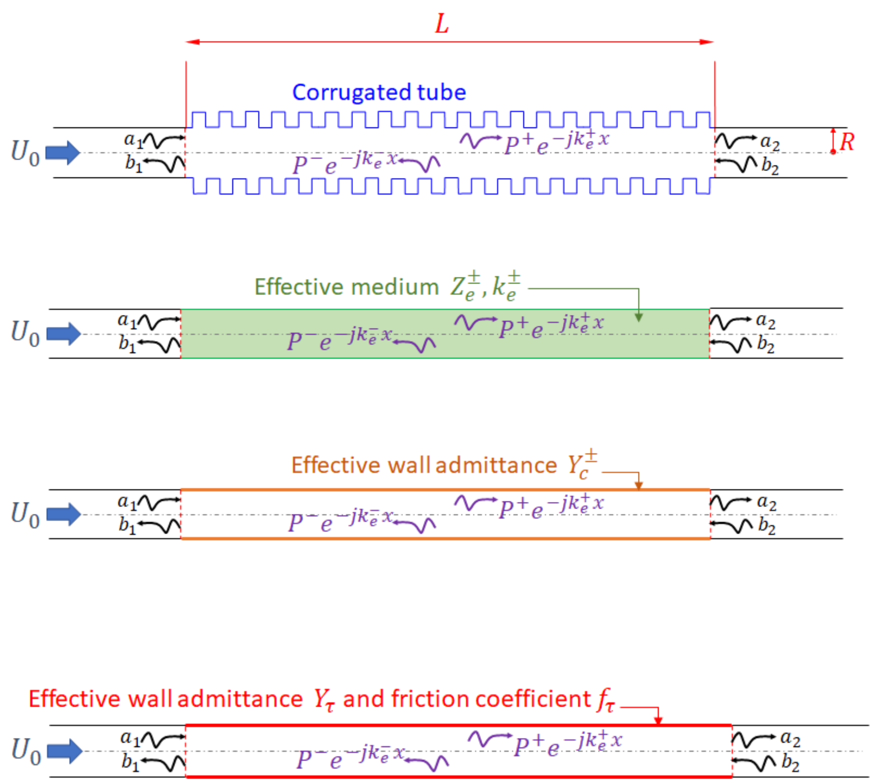

Fig. 1 Modeling of a corrugated axisymmetrical pipe (top sketch) as an equivalent material (middle sketch) or as equivalent wall admittances (lower sketches).

equation in presence of a uniform flow is considered in a cylindrical tube of diameter R:

$$\Delta_\perp p + \alpha^2 p = 0, \tag{1}$$

where

$$\Delta_\perp = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} \tag{2}$$

and

$$\alpha^2 = \Omega^2 - k^2 \quad \text{with} \quad \Omega = \frac{\omega}{c_0} - M_0 k. \tag{3}$$

Taking $p = G(r)e^{-jkx}$ the θ-independent pressure leads to

$$\frac{\partial^2 G}{\partial r^2} + \frac{1}{r}\frac{\partial G}{\partial r} = -\alpha^2 G \tag{4}$$

for which $G(r) = P_0 J_0(\alpha r)$ is solution. The acoustic radial velocity is then written using Euler equation

$$-\frac{\partial p}{\partial r} = j\Omega\rho_0 c_0 v. \tag{5}$$

Within the thin boundary layer, the velocity decreases and vanishes at the (equivalent) wall position. The Ingard-Myers condition is used as boundary condition at the treated wall, imposing continuity of the radial displacement leading to:

$$v(R) = \frac{\omega - u_0 k}{\omega} v_m, \tag{6}$$

and continuity of pressure $p(R) = p_m$, where p_m and v_m are the pressure and velocity outside of the boundary layer, linked by $Y_c p_m = v_m$ to an admittance Y_c equivalent to the corrugations. The equivalent admittance is computed by inserting the solution of equation 4 in the Euler equation (5) and evaluating it at $r = R$ using the Ingard-Myers boundary condition (6). This leads to:

$$Y_c = -jR\frac{k_0^2}{\Omega^2}\frac{\alpha J_1(\alpha r)}{J_0(\alpha r)}. \tag{7}$$

As will be seen in section IV, the measured wavenumbers are not identical for the positive and negative propagation directions. Equation 7 is thus written for both directions:

$$Y_c^{\pm} = -jR\frac{k_0^2}{\Omega^{\pm 2}}\frac{\alpha^{\pm} J_1(\alpha^{\pm} r)}{J_0(\alpha^{\pm} r)} \quad \text{with} \quad \alpha^{\pm} = \sqrt{\Omega^{\pm 2} - k_e^{\pm 2}} \quad \text{and} \quad \Omega^{\pm} = \frac{\omega}{c_0} \mp M_0 k_e^{\pm}. \tag{8}$$

This expression allows the computation of the impedance equivalent to the corrugations for the two flow directions.

B. The stress-impedance model

In the classical definition of the wall impedance for a locally-reacting wall, the admittance measured for acoustic propagation in both directions should be identical. However, it has been observed that the impedance (or admittance) could differ depending on the propagation direction [6]. Diverse works have been done to explain this observed difference [6–8]. Here, we will use the concept of Stress-Impedance, adapted from [5] to model the interaction between the flow in the innermost layer close to the wall and the acoustic wave.

The principle is to write the direction-dependent admittance $Y_c^{\pm}$ as the sum of a unique admittance Y_τ and of a contribution from the unsteady transfer of momentum from the flow into the wall due to the cavities and to turbulence:

$$Y_c^{\pm} = Y_\tau + \frac{k_e \pm}{\omega} f_\tau, \tag{9}$$

with f_τ a friction depending on depending on the pressure at the wall.

From the two different values measured $Y_c^{\pm}$, we can compute Y_τ and f_τ:

$$f_\tau = \frac{k_0}{k_e^+ - k_e^-}(Y_c^+ - Y_c^-) \quad \text{and} \quad Y_\tau = Y_c^+ - \frac{k_e^+}{k_0} f^\tau. \tag{10}$$

III. Experimental setup and methodology

The experimental setup is similar to that described in [3] The pipe to be investigated is mounted between two measurement pipes of diameter 30 mm equipped each with 3 microphones. On each side, an acoustic source and an anechoic termination are mounted, and a main flow is imposed by a centrifugal fan located upstream. The flow velocity and temperature are measured upstream of the measurement section.

A corrugated pipe of inner diameter $D = 2R = 30$ mm is used for the experiments. The corrugations are axisymmetric cavities of depth 4 mm and width 4 mm, with a rounding of radius 1 mm of the upstream edge and a sharp downstream edge. The cavities are regularly spaced with a constant pitch of 12 mm. To allow flexibility, the pipe is made of threaded rings which can be assembled together to form a pipe of length up to 2 m. The experiments reported here are performed with a length $L = 1.980$ m. A sketch of the pipe is presented in Figure 2.

For each configuration (and at each flow velocity) the transfer matrix is measured in two steps: First, the acoustic field (a step-sine) is created by the sound source located upstream, while the mouth of the downstream sound source to the pipe is closed. Second, the acoustic field is created by the sound source located downstream, while the mouth of the upstream sound source to the pipe is closed. During each measurement, the sound wave incident on the test object and the reflected sound waves are reconstructed from the microphones signals. From these two measurements, the four coefficients of the diffraction matrix can thus be computed.

The wavenumbers corresponding to the acoustic propagation in forward and backward directions in the corrugated pipe is then computed from the coefficients of the diffraction matrix with a continuous model [4] of the corrugated pipe and of the smooth measurement pipes.

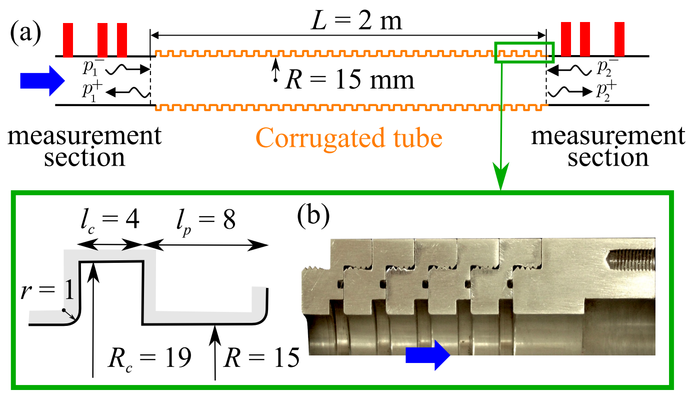

Fig. 2 Sketch of the corrugated pipe used for the experiments.

IV. Results

The results presented here correspond to measurements done without main flow and with air flows at $M = 0.016$ and $M = 0.03$ in the corrugated pipe. First, the wavenumber in the corrugated pipe will be displayed for the different flow velocities, then the corresponding equivalent wall admittance are computed and discussed. The equivalent wall admittance is first computed using the Ingard-Myers condition, then with the stress-impedance model.

In the left plot of Figure 3, the wavenumbers measured in the positive and negative directions without flow and with a mean flow of $M = 0.016$ and $M = 0.03$ are compared. The wavenumber is here divided by the reference wavenumber $k_0 = \omega/c_0$. The first feature to be observed is that the real part of the wavenumber is higher than k_0, which means that acoustic waves propagating in the corrugated section travel at a lower velocity than the velocity of sound due to the additional compressibility due to the cavities [1]. The admittance equivalent to the corrugations computed from the measured wavenumbers (using the Ingard-Myers condition) with Equation 8 is also displayed on the right of Figure 3. A number of interesting features are visible on these figures:

- The real part of the wavenumber depends on the flow (and on the direction): $Re(k_e^+) > Re(k_e^{M=0} > Re(k_e^-)$, which is due to the convection;
- Strong oscillations are observed on the wavenumber. These oscillations occur at roughly constant Strouhal number and are typical flow-acoustic coupling. Furthermore, an interesting feature of the result presented in this figure is the oscillations are so large that the imaginary part of the wavenumber is negative for a certain frequency range (around 550 Hz at $M = 0.016$ and around 950 Hz at $M = 0.03$, which corresponds to $Sr \approx 0.5$). This means that the sound waves at these frequencies are amplified when traveling through the corrugated pipe when there is a mean flow. This amplification occurs for waves traveling in both directions, but slightly more for waves traveling against the main flow. This is this amplification which can be the cause of whistling, if the resonator consisting of the corrugated pipe and its boundary conditions is strong enough, that is, if the acoustic losses are smaller than the acoustic gain due the corrugations;
- The same oscillations are observed on the equivalent wall admittance. Here, the amplification of sound by the wall translates as a negative conductance.
- Since the impedance (or admittance) is a property of the material, it should be independent of the flow direction. The same value should thus be found whether M_0 is positive or negative. However, it is found that the results are different if the positive-direction wavenumber k^+ or if the negative-direction wavenumber k^- are used. The admittance computed from k^+ (continous blue and red lines) are different from the admittance computed from k^- (dashed red and blue lines). Contrary to the case of the impedance of liner, this difference is not more or less identical for all frequencies, but is mainly observed at the frequencies where the flow-acoustic coupling is the most important.

The wall admittance as defined using the Stress-Impedance model is computed with Equation 10 and plotted in Figure 4.The wall admittance presents the same general shape and has the same features as the admittance computed without the Stress-Impedance model (Sr-dependent oscillations, negative conductance, ...) but this quantity is now uniquely defined. The corresponding stress factor is also plotted in Figure 4. The amplitude of the stress factor shows a bump in the region of maximum absorption. Furthermore, the phase of the stress factor shows a roughly constant slope for the low frequencies. The corresponding delay is of the same order as the travel time of a particle convected above

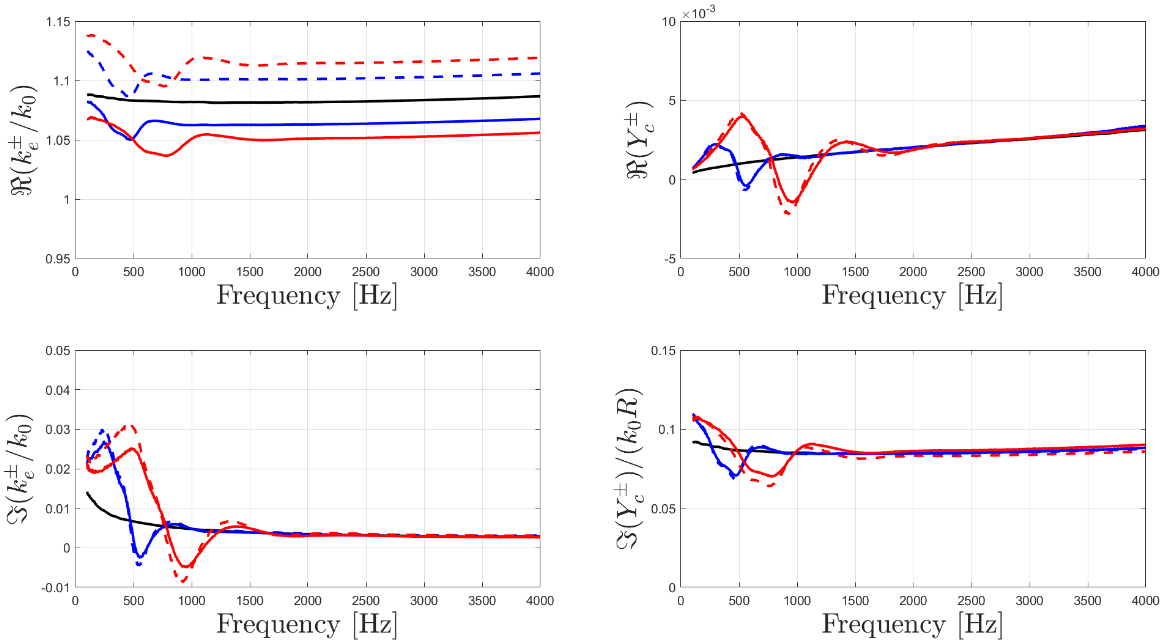

Fig. 3 Comparison of the wavenumbers k^- (left figure) and equivalent wall admittance (right figure) measured without flow (black line), at M=0.016 (blue lines) and M=0.03 (red lines). Plain lines: k_e^+ and Y_c^+ (measured for propagation in same direction as main flow); Dashed line: k_e^- and Y_c^- (measured for propagation in direction opposite to main flow).

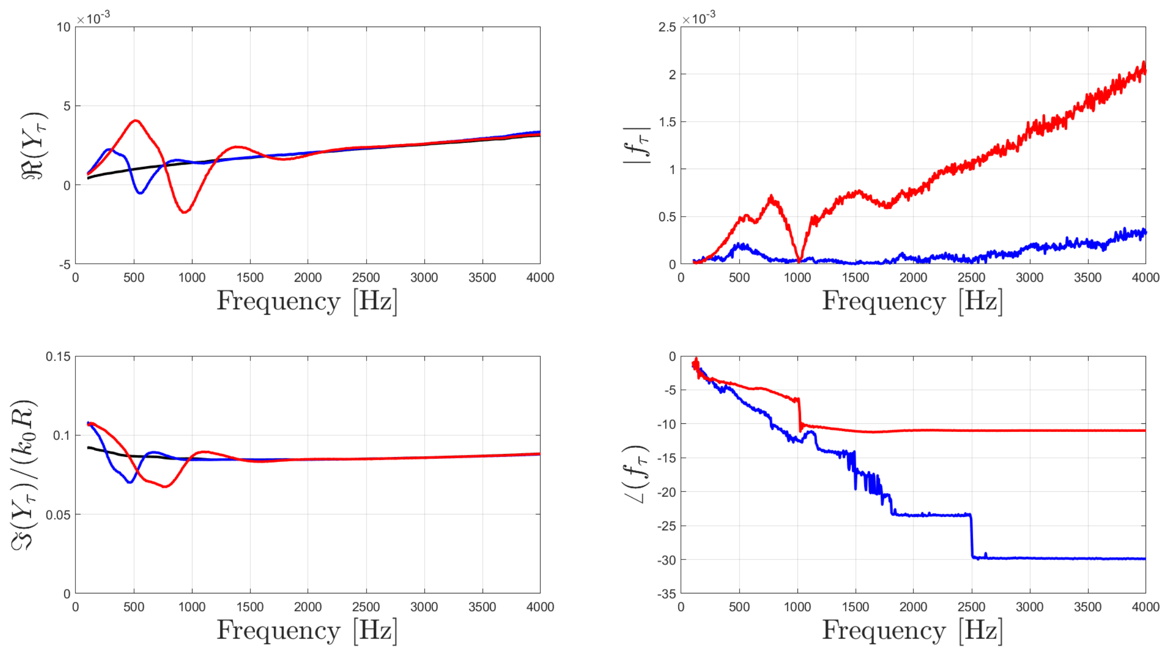

Fig. 4 Equivalent admittance of the corrugations computed with the stress-impedance model, measured without flow (black line), at M=0.016 (blue lines) and M=0.03 (red lines). Left plot: Equivalent impedance, Right plot: Equivalent friction factor

the opening of the cavities. This indicates that, in the case of the wall admittance equivalent to the corrugations the friction factor is possibly driven by the vortex shedding above the cavities.

V. Conclusions

A corrugated pipe can be described as a pipe with a equivalent wall treatment. The equivalent admittance of the treatment is computed from the measured acoustic wavenumber in the pipe. The equivalent admittance shows an oscillatory behavior correspond to the flow-acoustic interaction due to the vorticity shed above the cavities. For a certain frequency range (related to the main-flow velocity by a constant Strouhal number), the real part of the admittance is even negative, indicating sound production by the equivalent wall. As for the impedance of liners measured in the presence of mean flow, it is found that the equivalent wall admittance computed from the results of the measurement with acoustic wave traveling in the same direction as the flow is slightly different from that measured in the opposite direction. This difference is principally observed in the frequency range of maximum interaction. The Stress-Impedance model allows to describe the corrugation as an admittance which is flow-direction independent and a friction factor accounting for the difference.

References

[1] Nakiboğlu, G., Belfroid, S., Golliard, J., and Hirschberg, A., "On the whistling of corrugated pipes: effect of pipe length and flow profile," *Journal of Fluid Mechanics*, Vol. 672, 2011, pp. 78–108.

[2] Rudenko, O., Nakiboğlu, G., Holten, A., and Hirschberg, A., "On whistling of pipes with a corrugated segment: Experiment and theory," *Journal of Sound and Vibration*, Vol. 332, No. 26, 2013, pp. 7226 – 7242.

[3] Golliard, J., Sanna, F., Violato, D., and Auregan, Y., "Measured Source Term in Corrugated Pipes with Flow. Effect of Diameter on Pulsation Source," *Proceedings of the 22nd AIAA/CEAS Aeroacoustics Conference, 2016, Lyon, France*, 2016.

[4] Leroux, M., "Propagation acoustique en conduit traitée : influence de l'écoulement sur la propagation avec impédance de paroi," Ph.D. thesis, Université du Maine, Le Mans, 2005. Thèse de doctorat dirigée par Aurégan, Yves. Acoustique Le Mans 2005.

[5] Aurégan, Y., "On the use of a stress–impedance model to describe sound propagation in a lined duct with grazing flow," *Journal of the Acoustical Society of America*, Vol. 143, 2018. In press.

[6] Renou, Y., and Aurégan, Y., "Failure of the Ingard-Myers boundary condition for a lined duct: An experimental investigation," *Journal of the Acoustical Society of America*, Vol. 130, No. 1, 2011, pp. 52–60.

[7] Weng, C., Schulz, A., Ronneberger, D., Enghardt, L., and Bake, F., "Impedance eduction in the presence of turbulent shear flow using the linearized Navier-Stokes equations," *Proceedings of the 23nd AIAA/CEAS Aeroacoustics Conference, 2017, Denvers, CO*, 2017.

[8] Boden, H., Cordioli, J. A., Spillere, A., and Serrano, P., "Comparison of the effect of flow direction on liner impedance using different measurement methods," *Proceedings of the 23nd AIAA/CEAS Aeroacoustics Conference, 2017, Denvers, CO*, 2017.